

Oscillations and Waves



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1.1 Simple Harmonic Motion

1.1.1 Introduction

The Periodic motion, from mosses on springs to vibrations of atoms at lattice points, is one of the most important kinds of physical behaviour. This world is filled with oscillations in which objects move back and forth repeatedly. Here are a few examples: The motion of a pendulum in a wall clock, i.e. pendulum moves back and forth. When wind blows past a tree, the branches of trees oscillate so severely that it rips apart. When an airplane is in flight, the turbulence of the air flowing past the wings makes them to oscillate, eventually leading to metal fatigue and even failure. When we try to ride a bike with punctured tyre, its wheels oscillate horizontally (“hunt” in mechanical engineering terms) as they are forced to turn in new directions (you can hear the oscillations). When an earthquake occurs near surround a city, buildings may set oscillating so severely that they are shaken apart. The oscillations of electrical signals in electronic circuits (LC, LCR Circuits) and many more...

In this chapter we try to look in detail on Hooke's law and its applications, where the force is proportional to the displacement, which tends to restore objects to some equilibrium position. Many physical systems can be successfully studied and modelled with this simple idea, including the vibrations of strings, the swinging of a pendulum, and the propagation of waves in a medium of all kinds. In all these physical systems the basic phenomena involved is periodic motion. Periodic vibrations can cause disturbances that move through a medium in the form of waves. There are many kinds of waves occur in nature, such as sound waves, water waves, seismic waves, and electromagnetic waves. These very different physical phenomena are described by common terms and concepts in the chapter and we discuss a basic type of oscillation called simple harmonic motion.

1.1.2 Oscillation: “The periodic motion that occurs when a time varying force acts on an object”. If the oscillations occur without the action of external force then, such oscillations are called as *free oscillations*. If the oscillations occur with application of external force then, such oscillations are called as *forced oscillations*.

1.1.3 Simple Harmonic Motion is a special kind of periodic oscillation. The motion of an object is said to be SHM if the restoring force or acceleration is directly proportional to the displacement and it acts in the direction opposite to that of motion. Ex. Oscillations of simple pendulum, oscillations of weight attached to spring.

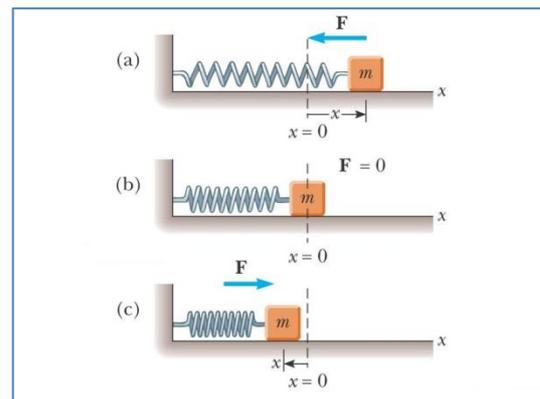
1.1.4 Expression for Simple Harmonic motion

Let us assume the object moves on a frictionless horizontal surface. If the spring is stretched or compressed a small distance “x” from its upstretched or equilibrium position and then released, it exerts a force on the object as shown in the figure.

∴ The force is directly proportional to displacement.

$$F \propto -x$$

$$F = -kx \text{ --- (1)}$$



This equation is called as Hooke's law and represents the Restoring force.

Where “k” is spring constant and its value is depends on stiffness of the spring.

The acceleration of an object moving with simple harmonic motion can be found by using Hooke's law in the equation for Newton's second law.

$$F = m \frac{d^2x}{dt^2}$$

$$\frac{d^2x}{dt^2} = \frac{-k}{m} x \text{ --- (2)}$$

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0 \text{ --- (3)}$$

This equation of motion is called as Simple Harmonic Oscillator (SHO) equation, for an object executing SHM. It is a second order linear differential equation in which the second derivative of dependent variable is proportional to the negative of dependent variable. And this is the basic equation for SHM.

Let $\frac{k}{m} = \omega^2$, Then we get

$$\frac{d^2x}{dt^2} + \omega^2x = 0$$

Where ' ω ' is called as angular frequency of oscillations.

1.1.5 Characteristics of Simple Harmonic Oscillations: An object with Simple Harmonic Motion has following properties.

- i. It has periodic motion about an equilibrium point.
- ii. The period of each cycle is constant.
- iii. For every period it has maximum displacement.
- iv. The Force is directly proportional to the displacement or acceleration.
- v. The periodic motion due to Force is always directed towards the equilibrium point.

1.1.6 Mechanical Simple Harmonic Oscillators (mass suspended to spring)

Consider an object of mass 'm' attached to spring with rigid support as shown in the figure. The object is made to oscillate up and down and these oscillations are due to restoring force in the spring. This force (F) is directly proportional to the displacement (y) from the equilibrium position.

According to Hooke's law,

$$F = -ky$$

Where 'k' is the spring constant and 'y' is the displacement of the mass on y – direction.

From Newton's second law we have

$$F = m \frac{d^2y}{dt^2}$$

On equating above equation, we get

$$\frac{d^2y}{dt^2} = \frac{-k}{m}y$$

$$\frac{d^2y}{dt^2} + \omega^2y = 0$$

Where ' ω ' is the angular frequency of oscillation, the auxiliary equation of above equation is

$$(D^2 + \omega^2)y = 0$$

The general solution of 'y' is given by

$$y = C_1 \cos \omega t + C_2 \sin \omega t$$

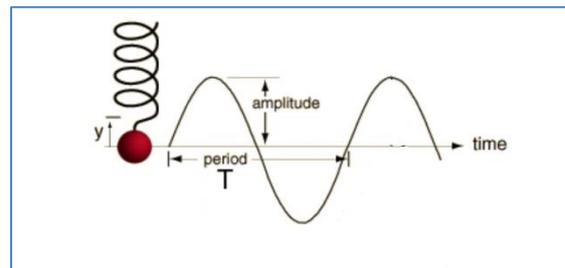
Where ' C_1 ' and ' C_2 ' are constants and its values depends on initial conditions.

The above equation reduced to the form

$$y = A \sin(\omega t + \phi) \text{ --- (1)}$$

Where 'A' is the amplitude and ' ϕ ' is the initial phase.

This is the equation for the displacement of the object in y – direction.



Velocity (v): The velocity of the object executing simple harmonic oscillations is the differential equation of displacement 'y' with respect to 't'.

$$v = \frac{dy}{dt} = A\omega \cos(\omega t + \phi)$$

The velocity of object during SHM varies from $+A\omega$ to $-A\omega$. The velocity of the object is zero for the maximum displacement and the velocity become Maximum for zero displacement of object on periodic motion.

Acceleration (a): The acceleration of a body in SHM is the change of velocity (v) with time (t) and it is given by the differential equation of 'v' with respect to time 't'.

$$a = \frac{d^2y}{dt^2} = -A\omega^2 \sin(\omega t + \phi) = -\omega^2 y$$

From the above equation acceleration is directly proportional to displacement and directed opposite to the displacement. Varies from $+A\omega^2$ to $-A\omega^2$.

Amplitude (A): The maximum displacement of the object during the periodic motion is called the amplitude (A) and its value varies from +A to -A.

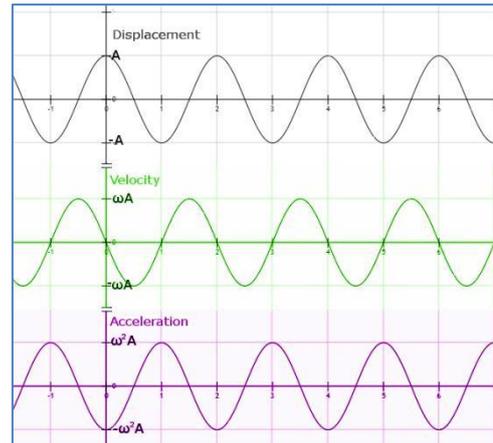
Phase Angle and Initial Phase: The quantity $(\omega t + \phi)$ is the phase of the periodic motion of the object and is called as phase angle. The angle ' ϕ ' is its initial phase.

Angular frequency or velocity: It is the rate of change of angular displacement and is given by

$$\omega = \sqrt{\frac{k}{m}}$$

Frequency: It is defined as number of oscillations per second and given by $f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$

Time period (T): The time taken for one complete oscillation and it is given by $T = \frac{1}{f}$



1.1.7 Mass Suspended Springs in Series and Parallel

Consider the two springs of spring constants ' k_1 ' and ' k_2 ' respectively, with suspended mass ' m '. Let ' k_s ' be the effective spring constant when two springs in series and ' k_p ' be the effective spring constant for two springs in parallel as shown in figure.

Springs in Series

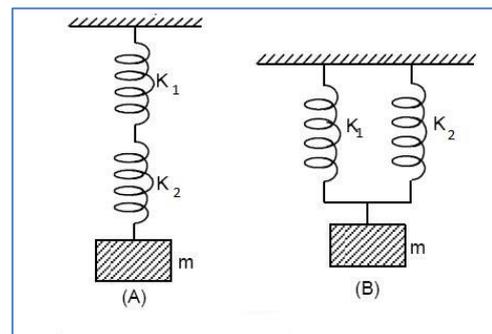
When springs are in connected in series to the extension is the net displacement 'y' is given by

$$y = y_1 + y_2 \text{ --- --- (1)}$$

In the above equation ' y_1 ' is the displacement due to first spring with restoring force ' F_1 ' and ' y_2 ' is the displacement due to second spring with restoring force ' F_2 ' respectively. If ' F ' is the effective restoring force due to applied mass ' m '.

$$\begin{aligned} \text{Then we have } F_1 &= k_1 y_1 & \text{or} & & y_1 &= \frac{F_1}{k_1} \\ F_2 &= k_2 y_2 & \text{or} & & y_2 &= \frac{F_2}{k_2} \\ F &= k_s y & \text{or} & & y &= \frac{F}{k_s} \end{aligned}$$

Substituting in (1), we get



$$\frac{1}{k_s} = \frac{1}{k_1} + \frac{1}{k_2}$$

The effective spring constant or force constant of the series combination is given by

$$k_s = \frac{k_1 k_2}{k_1 + k_2}$$

Springs in parallel

In this case, the displacement 'y' due to both springs of force 'F₁' and 'F₂' respectively. The effective restoring force 'F = k_py' of the system is given by

$$F = F_1 + F_2$$

$$k_p y = k_1 y + k_2 y$$

The effective spring constant is

$$k_p = k_1 + k_2$$

1.1.8 Complex notation and Phasor representation of SHM

The periodic motion of a system can be described by complex notation. In general the complex notation can be represented by

$$z = x + iy.$$

It has two components 'x' is a real part of 'z' and 'y' is the imaginary part of 'z'.

The complex conjugate of 'z' is $z^* = x - iy$

The product of 'z' and 'z*' is $zz^* = x^2 + y^2$

These components are represented by Argand diagram.

The angle 'φ' is the angle between 'OX' and 'OZ' measured in anticlockwise.

$$\text{Cos } \phi = \frac{x}{r} \quad \text{and} \quad \text{Sin } \phi = \frac{y}{r}$$

Where $r = |z| = \sqrt{x^2 + y^2}$

The displacement of oscillatory motion can be represented by

$$z = Ae^{i\phi}$$

But $e^{i\phi} = \text{Cos } \phi + i \text{Sin } \phi$

$$z = x + iy = A(\text{Cos } \phi + i \text{Sin } \phi) \text{ --- (1)}$$

The Equation for the Simple Harmonic Motion is

$$\frac{d^2 z}{dt^2} + \omega^2 z = 0$$

The Auxiliary equation is

$$(D^2 + \omega^2)z = 0$$

The solution for displacement is $z = C \text{Cos } \omega t + D \text{Sin } \omega t \text{ --- (2)}$

The reduced form of displacement is $z = A \text{Sin } (\omega t + \phi) \text{ --- (3)}$

On equating (2) and (3), we get

$$A \text{Sin } (\omega t + \phi) = C \text{Cos } \omega t + D \text{Sin } \omega t \text{ --- (4)}$$

We have $A \text{Sin } (\omega t + \phi) = (A \text{Sin } \omega t \cdot \text{Cos } \phi + A \text{Cos } \omega t \cdot \text{Sin } \phi) \text{ --- (5)}$

On comparing (4) and (5)

$$A \text{Cos } \phi = D \quad \text{and} \quad A \text{Sin } \phi = C$$

Squaring and adding above equation, we get

$$A^2 \text{Cos}^2 \phi + A^2 \text{Sin}^2 \phi = C^2 + D^2$$

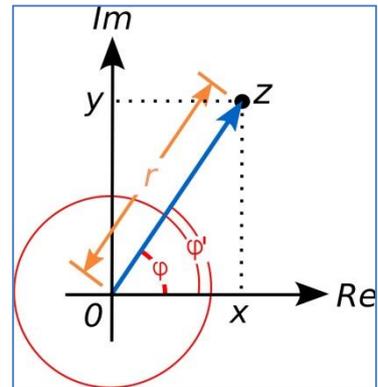
$$A^2 = C^2 + D^2$$

$$A = \sqrt{C^2 + D^2}$$

Dividing the Equations, we get

$$\phi = \tan^{-1} \left[\frac{C}{D} \right]$$

This is the angle between x – axis and resultant 'z'



1.1.9 Natural frequency of oscillations: When a body exhibits free oscillations the frequency with which the oscillations occur is called Natural Frequency.

1.2.1 Damped Oscillations: For an object with periodic oscillations, the amplitude of oscillations decreases with respect to time. **The decrease in amplitude of the oscillations may be due to the involvement of**

resistive forces like friction. Hence the amplitude of damped oscillations decreases continuously and finally comes to rest position. The following are the examples for damped oscillations.

- i. A simple pendulum oscillating in air. During the motion the pendulum experiences air resistance which leads to the dissipation of energy. Hence the amplitude of oscillations of the simple pendulum decreases and finally comes to a stop.
- ii. An object of mass 'm' suspended in liquid with spring. During the oscillations the viscous force acting on the mass reduces the amplitude gradually.

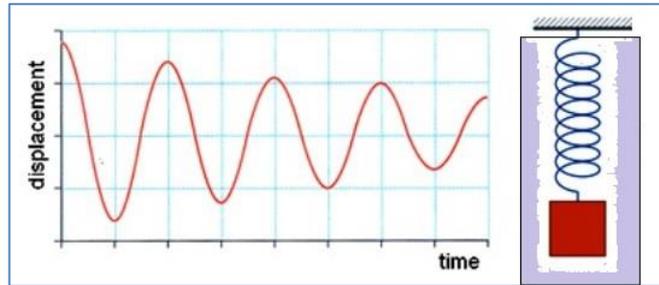
1.2.2 Theory of Damped Oscillations

Expression for decay of the amplitude in damped oscillations.

Consider a Simple Harmonic Oscillator with decreasing in the amplitude of oscillations. The Restoring force is the sum of force due to displacement and change in displacement with time (velocity) within resistive medium and it is given by.

$$F = -ky - b \frac{dy}{dt}$$

In the above equation 'k' is the spring constant, and 'b' is a constant depends on resistive medium, shape and size of the pendulum. Now by newton's second law



$$F = m \frac{d^2y}{dt^2}$$

On equating the above equations, we get.

$$m \frac{d^2y}{dt^2} = -ky - b \frac{dy}{dt}$$

$$\frac{d^2y}{dt^2} + \frac{b}{m} \frac{dy}{dt} + \frac{k}{m} y = 0 \text{ --- (1)}$$

Let $\frac{k}{m} = \omega_0^2$ is the natural frequency of oscillations and $\frac{b}{m} = \gamma$ is the damping coefficient, it depend on the medium. The auxiliary equation of above equation is.

$$(D^2 + \gamma D + \omega_0^2)y = 0$$

On solving, $(D^2 + \gamma D + \omega_0^2) = 0$ and $y \neq 0$

The roots of the above equation is

$$D_1 = \frac{-b}{2m} + \frac{1}{2m} \sqrt{b^2 - 4mk}$$

$$D_2 = \frac{-b}{2m} - \frac{1}{2m} \sqrt{b^2 - 4mk}$$

The general equation for (1) is

$$y(t) = M e^{\left(\frac{-b}{2m} + \frac{1}{2m} \sqrt{b^2 - 4mk}\right)t} + N e^{\left(\frac{-b}{2m} - \frac{1}{2m} \sqrt{b^2 - 4mk}\right)t}$$

For small damping, the above equation is reduced to

$$y(t) = A e^{\frac{-b}{2m}t} \text{Cos}(\omega t + \phi)$$

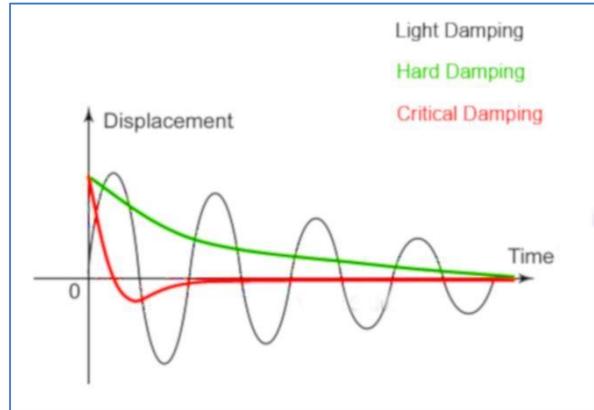
Where ' ω ' is the frequency of damped oscillations and it is given by

$$\omega = \sqrt{\omega_0^2 + \left[\frac{-b}{2m}\right]^2}$$

The coefficient of Cosine term gives amplitude of the damped oscillations and from above equation, it is given by $Ae^{-\frac{b}{2m}t}$. Therefore the amplitude of oscillations decreases exponentially, and it is less than that of natural frequency ω_o .

The damped oscillations are classified as follows.

- a. **Under damped Oscillations:** The oscillations are said to be under damped or light damped or weakly damped, if the retarding force is weaker than the restoring force. Then the amplitude of the oscillations decreases with respect to time. The condition for damped oscillations is $b^2 < 4mk$.
- b. **Over damped Oscillations:** The oscillations are said to be over damped or hard damped, if system attains equilibrium state slowly without making any oscillations. The condition for the damped oscillations is $b^2 > 4mk$.
- c. **Critical Damped oscillations:** When a system attains equilibrium state without making any oscillations is called critical oscillations. The condition for the damped oscillations is $b^2 = 4mk$. The below figure shows the all the three types of oscillations.



Quality factor: It is defined as 2π times the ratio of energy stored in an oscillation to the energy lost per time period.

$$Q = 2\pi \frac{E}{PT}$$

$$Q = \omega\tau$$

Where $\omega = \frac{2\pi}{T}$ is the angular frequency and ' τ ' is the relaxation time $\tau = 1/\gamma$ and it is defined as the time taken for an oscillator to decay the energy to $1/e$ times of its initial value. It is also given by.

$$Q = \frac{1}{\gamma} \sqrt{\frac{k}{m}}$$

1.2.3 Theory of Forced Oscillations

'The oscillations occur that under the action of an external periodic force are called forced oscillations'. During forced oscillations the system oscillates with the frequency of the external periodic force. Examples for forced oscillations are Sonometer wire set to oscillations using a tuning fork and Resonance air column.

i. Expression for Amplitude and Phase in Forced Oscillations

The forces acting on the system during forced oscillations

- a. Restoring force acting in the direction opposite to the displacement.
- b. Damping force due to resistive medium.
- c. External periodic force acting on the system.

Thus the equation in differential form could be written as

$$m \frac{d^2y}{dt^2} = -ky - b \frac{dy}{dt} + F_o \text{Cos}\omega t$$

$$\frac{d^2y}{dt^2} + \frac{b}{m} \frac{dy}{dt} + \frac{k}{m} y = \frac{F_o}{m} \text{Cos}\omega t$$

$$\frac{d^2y}{dt^2} + \gamma \frac{dy}{dt} + \omega_o^2 y = \frac{F_o}{m} \text{Cos}\omega t$$

In the above equation $\gamma = \frac{b}{m}$ called as damping coefficient, $\omega_o = \sqrt{\frac{k}{m}}$ is the natural frequency, $\frac{F_o}{m} \text{Cos}\omega t$ is the applied period force and ω is the frequency of applied periodic force. The above equation could be represented in complex exponential form as.

$$\frac{d^2z}{dt^2} + \gamma \frac{dz}{dt} + \omega_o^2 z = \frac{F_o}{m} e^{i\omega t} \text{----- (1)}$$

The solution for this equation is

$$z = A e^{i(\omega t - \phi)} \text{----- (2)}$$

Substituting Equation (2) in (1), we get

$$(-\omega^2 A + i\omega\gamma A + \omega_0^2 A)e^{i(\omega t - \varphi)} = \frac{F_0}{m} e^{i\omega t}$$

$$(\omega_0^2 - \omega^2)A + i\omega\gamma A = \frac{F_0}{m} e^{i\omega t} e^{-i(\omega t - \varphi)}$$

$$(\omega_0^2 - \omega^2)A + i\omega\gamma A = \frac{F_0}{m} e^{i\varphi}$$

The above equation could be written as

$$(\omega_0^2 - \omega^2)A + i\omega\gamma A = \frac{F_0}{m} (\cos\varphi + i\sin\varphi)$$

Equating the real and imaginary parts on LHS and RHS in above equation

$$(\omega_0^2 - \omega^2)A = \frac{F_0}{m} \cos\varphi \text{ ----- (3)}$$

$$i\omega\gamma A = \frac{F_0}{m} \sin\varphi \text{ ----- (4)}$$

Squaring and adding the equations (3) and (4), we get Amplitude as

$$A = \frac{F_0}{m\sqrt{(\omega_0^2 - \omega^2)^2 + (\gamma\omega)^2}}$$

Dividing equation (4) by (3), we get phase of the oscillations as

$$\tan\varphi = \frac{\gamma\omega}{\omega_0^2 - \omega^2}$$

Conditions for forced Oscillations:

- i. If $\omega \ll \omega_0$ i.e. if the frequency of the applied frequency is less than the natural frequency, The amplitude of the forced oscillations is

$$A = \frac{F_0}{m\omega_0^2}$$

From the above equation the amplitude is independent of ' ω ' even though the system oscillates with the applied frequency ' ω '. The phase of the oscillations is given by

$$\tan\varphi = \frac{\gamma\omega}{\omega_0^2} \approx 0$$

Thus the displacement and phase are almost in phase.

- ii. If $\omega = \omega_0$ The amplitude becomes

$$A = \frac{F_0}{m\gamma\omega_0}$$

The amplitude of the oscillations is maximum; this condition is called the *Resonance*. The corresponding phase difference is

$$\tan\varphi = \infty$$

Hence the phase difference between displacement and applied force is $\frac{\pi}{2}$

- iii. If $\omega \gg \omega_0$ This case is significant only if the damping force is very small (for small γ). The amplitude is

$$A = \frac{F_0}{m\sqrt{(\omega^2)^2 + (\gamma\omega)^2}}$$

And the Phase is

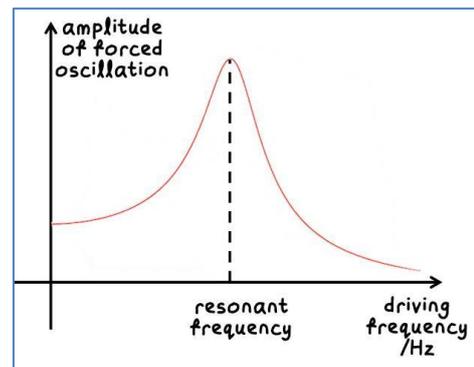
$$\tan\varphi = -\frac{\gamma}{\omega}$$

For small γ

$$\tan\varphi = 0$$

The phase difference between the displacement and applied force is ' π '.

The above three cases are shown in figure.



1.2.4 Resonance

When a system under the forced oscillations in which the frequency of the applied force is varied. If the frequency (ω) of the applied periodic force matches with the natural frequency (ω_0) of oscillations of the system, then the amplitude of the oscillations will be maximum results in maximum transfer of energy occurs from driving system to the oscillating system. At resonance the system will be in phase with the applied periodic force. The amplitude is

$$A = \frac{F_0}{m\gamma\omega}$$

Examples of Resonance

The following are the examples of resonance in different oscillating systems under forced oscillations.

- i. In Sonometer when the natural frequency of the stretched string is equal to the frequency of the tuning fork the amplitude of oscillation is maximum.
- ii. Helmholtz resonator
- iii. Resonance in LCR circuits, an example for electrical resonance.
- iv. The absorption of energy by electrons in atoms.
- v. Resonance air column.

1.2.5 Sharpness of Resonance

The sharpness of resonance is the rate of change of amplitude with respect to a small change in frequency of the applied external periodic force, at resonance. Mathematically

$$\text{Sharpness of Resonance} = \frac{\Delta A}{\Delta \omega}$$

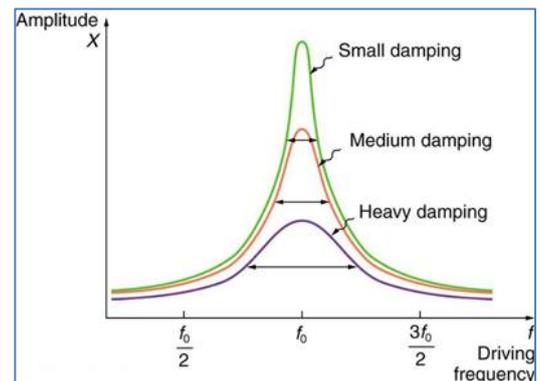
Where ' ΔA ' is the change in the amplitude to the corresponding change in frequency ' $\Delta \omega$ ' at resonance.

1.2.6 The Effect of damping on the sharpness of resonance.

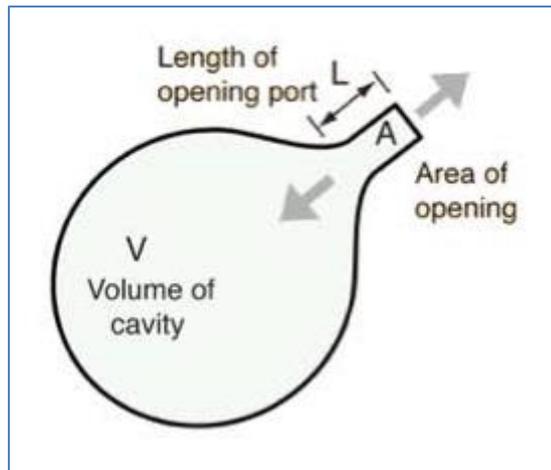
We have the equation for Sharpness of resonance is

$$\text{Sharpness} = \frac{F_0}{b\omega\omega_0}$$

During the tuning of oscillating system the rate at which the amplitude varies near resonance depends on damping. Smaller the damping sharper will be the resonance and larger the damping flatter will be resonance.



1.2.7 The Helmholtz Resonator: It is an important type of resonator with very different acoustic characteristics. It is made of a hollow sphere with a short and small diameter neck. The Helmholtz resonator has a single isolated resonant frequency. If (f) is resonant frequency of a classical Helmholtz resonator, and it is determined by its volume (V) and by the length (L) and area (A) of its neck.

**Working:**

When a resonator is held near the source of a sound, the air in it will begin to resonate if the tone being analysed, it has a spectral component at the frequency of the resonator. By listening carefully to the tone of a musical instrument with such a resonator, it is possible to identify the spectral components of a complex sound wave such as those generated by musical instruments. The expression for resonant frequency in Helmholtz resonator is given by

$$f = \frac{v}{2\pi} \sqrt{\frac{A}{VL}}$$

Numerical Problems:

1. A block whose mass m is 680gm is fastened to a string whose spring constant k is 65N/m. The block is pulled a distance $x=11$ cm from its equilibrium position at $x=0$ on a frictionless surface and released from rest at $t=0$. What are the angular frequency, frequency and period of the resulting motion?

SOLUTION: $m=680 \times 10^{-3}$ kg, $x=11$ cm, $k=65$ N/m

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{65}{680 \times 10^{-3}}} = 9.77 \text{ rad/sec}$$

$$f = \frac{\omega}{2\pi} = \frac{9.77}{2\pi} = 1.55 \text{ Hz}$$

$$T = \frac{1}{f} = \frac{1}{1.55} = 0.642 \text{ sec}$$

2. A 0.12Kg body undergoes SHM of amplitude 8.5 cm and a period 0.2 sec if the oscillations are produced by a spring, what is the spring constant?

SOLUTION: $m=0.12$ Kg, $A=0.085$ m, $T=0.2$ sec

$$T = \frac{1}{f} \Rightarrow f = \frac{1}{T} = \frac{1}{0.2} = 5 \text{ Hz}$$

$$\omega = 2\pi f = 2\pi \times 5 = 31.4 \text{ rad/sec}$$

$$\omega = \sqrt{\frac{k}{m}} \Rightarrow k = \omega^2 m = 31.4^2 \times 0.12 = 118.31 \text{ N/m}$$

3. A sewing machine needle moves along a path of 4cm long with an oscillating frequency of 10 Hz. Calculate the displacement at 1/120 second after crossing the centre of the path.

SOLUTION: linear path $2A=4\text{cm}$, amplitude $A=0.02\text{m}$, $f=10\text{Hz}$, $t=1/120\text{ sec}$

$$\omega = 2\pi f = 10 \times 2\pi = 62.8 \text{ rad/sec}$$

$$y = A \sin \omega t$$

$$y = 0.02 \sin(62.8 \times 1/120) = 9.99 \times 10^{-3} \text{m} = 0.999 \text{ cm}$$

4. A free particle is executing SHM in a straight line with a period of 25seconds, 5 seconds after it has crossed the equilibrium point, the velocity is found to be 0.7 m/s. Calculate the displacement at the end of 10 seconds and also the amplitude of oscillation.

SOLUTION: $T=25\text{ sec}$, At time $t= 5\text{ sec}$, velocity $v=0.7\text{m/s}$

Displacement, $y = A \sin \omega t$ under the absence of initial phase ϕ

$$\text{Angular frequency } \omega = \frac{2\pi}{T} = \frac{2\pi}{25} = 0.2512 \text{ rad/sec}$$

$$\text{And velocity } v = \frac{dy}{dx} = A\omega \cos \omega t \Rightarrow 0.7 = A \times 0.2512 \cos(0.2512 \times 5)$$

$$A=9.01 \text{ m}$$

Displacement is calculate after 10 sec,

$$y = A \sin \omega t \Rightarrow y = 9.01 \sin(0.2512 \times 10) = 5.30 \text{ m}$$

5. A 4 kg mass attached to a spring is observed to oscillate with a period of 2 seconds. Calculate the period of oscillation if a 6 Kg is attached to the spring.

SOLUTION: mass $m_1=4\text{ Kg}$, $T=2\text{ sec}$, $m_2=6\text{ Kg}$

$$\text{Time period } T = 2\pi \sqrt{\frac{m}{k}}$$

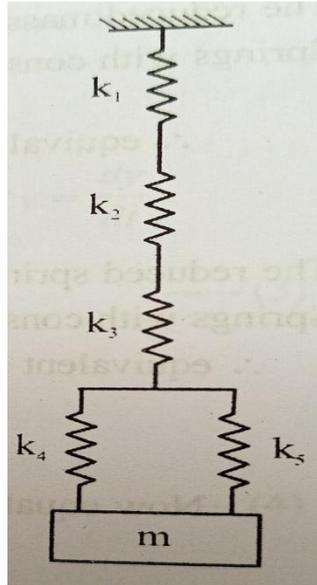
Squaring both sides and simplifying

$$k = \frac{(2\pi)^2 m}{T^2} = \frac{(2\pi)^2 4}{(2)^2} = 39.438 \text{N/m}$$

For the mass $m_2=6\text{ Kg}$,

$$T = 2\pi \sqrt{\frac{m}{k}} = T = 2\pi \sqrt{\frac{6}{39.438}} = 2.45 \text{ sec}$$

6. For the system in the figure, determine mass m such that the system has a natural frequency of 10 Hz. Taking $k_1=2000\text{ N/m}$, $k_2=1500\text{N/m}$, $k_3=3000\text{N/m}$, $k_4=k_5=500\text{ N/m}$.



SOLUTION: frequency $f=10$ Hz.

We know that frequency is given by $f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$ ----- (1)

To find the equivalent spring constant (k)

Here k_1, k_2, k_3 are connected in series

$$\frac{1}{k_s} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} = \frac{1}{2000} + \frac{1}{1500} + \frac{1}{3000} = 1.5 \times 10^{-3}$$

$$k_s = 666.67 \text{ N/m}$$

Again k_4, k_5 are connected in parallel

$$k_p = k_4 + k_5 = 500 + 500 = 1000 \text{ N/m}$$

Now again k_s and k_p are connected in series

$$\frac{1}{k} = \frac{1}{k_s} + \frac{1}{k_p} = \frac{1}{666.67} + \frac{1}{1000} = 2.49 \times 10^{-3}$$

$$k = 400 \text{ N/m}$$

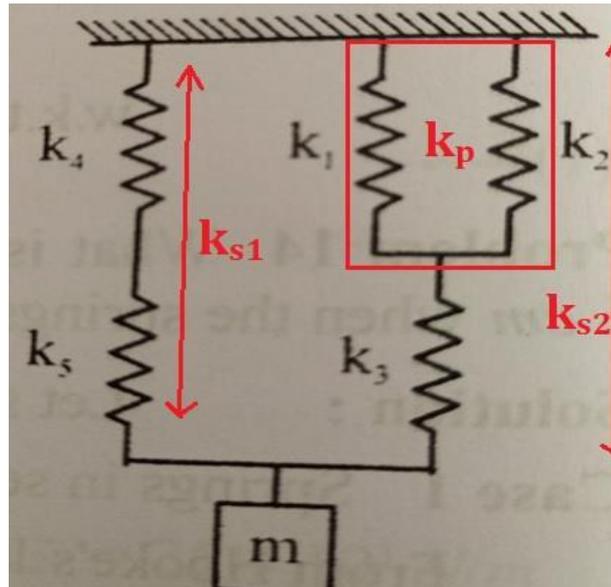
Now let us calculate mass

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

Squaring and simplifying

$$m = \frac{k}{(2\pi)^2 T^2} = \frac{400}{(2\pi)^2 10^2} = 0.1 \text{ Kg}$$

7. For the mass-spring system as shown in figure determine natural frequency if 2 Kg mass is attached to the system. Taking $k_1=500$ N/m, $k_2=1000$ N/m, $k_3=1500$ N/m, $k_4=2000$ N/m, $k_5=2500$ N/m.



SOLUTION: mass $m=2$ Kg.

We know that frequency is given by $f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$ ----- (1)

To find the equivalent spring constant (k)

Here k_4, k_5 are connected in series

$$\frac{1}{k_{s1}} = \frac{1}{k_4} + \frac{1}{k_5} = \frac{1}{2000} + \frac{1}{2500} = 9 \times 10^{-4}$$

$$k_{s1} = 1111.11 \text{ N/m}$$

Again k_1, k_2 are connected in parallel

$$k_p = k_1 + k_2 = 500 + 1000 = 1500 \text{ N/m}$$

Now again k_{s1} and k_3 are connected in series

$$\frac{1}{k_{s2}} = \frac{1}{k_{s1}} + \frac{1}{k_3} = \frac{1}{1111.11} + \frac{1}{1500} = 1.56 \times 10^{-3}$$

$$k_{s2} = 638.29 \text{ N/m}$$

Now again k_{s2} and k_p are connected in parallel

$$k = k_p + k_{s2} = 1500 + 639.29 = 2138.29 \text{ N/m}$$

Now let us calculate natural frequency

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = \frac{1}{2\pi} \sqrt{\frac{2138.29}{2}} = 5.2 \text{ Hz}$$

8. A free particle is executing simple harmonic motion in a straight line. The maximum velocity it attains during any oscillation is 62.8 m/s. find the frequency of oscillation, if its amplitude is 0.5 m.

Maximum velocity during oscillation, $V_{\max}=62.8$ m/s

Amplitude, $A=0.5$ m

We have equation for free vibrations,

$$x=a \sin \omega t$$

Velocity is given by

$$V = \frac{dx}{dt} = A\omega \cos \omega t = A\omega \sqrt{1 - \sin^2 \omega t} = \omega \sqrt{A^2 - A^2 \sin^2 \omega t} = \omega \sqrt{A^2 - x^2}$$

The particle attains maximum velocity while passing through its equilibrium position, at which time, the displacement is zero.

$$V_{max} = \omega\sqrt{A^2 - 0} = \omega A$$

Angular frequency,

$$\omega = \frac{V_{max}}{A} = \frac{62.8}{0.5} = 125.6 \text{ rad/sec}$$

Frequency of oscillation,

$$f = \frac{\omega}{2\pi} = \frac{125.6}{2\pi} = 20 \text{ Hz}$$

9. A mass of 0.5 Kg attached to a spring to set to vibrate. The vibrating motion is represented by the equation $\frac{1}{2} \frac{d^2y}{dt^2} + 0.014 \frac{dy}{dt} + 1.5 y = 0$. Calculate damping constant and angular frequency.

SOLUTION: $m=0.5$ Kg, this is the damped oscillation equation is given by

$$m \frac{d^2y}{dt^2} + ky + b \frac{dy}{dt} = 0$$

$b=0.014$, $k=1.5$

Damping constant, $\gamma = \frac{b}{m} = \frac{0.014}{0.5} = 0.028$

Angular frequency, $\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{1.5}{0.5}} = 1.73 \text{ rad/sec}$

10. The Q value of a spring loaded with 0.5 Kg is 75. The spring vibrates with a frequency of 5 Hz. Calculate the force constant and damping constant.

SOLUTION : $Q=75$, $m=0.5$ Kg, $f=5$ Hz

Frequency,

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

Squaring and simplifying

$$k = (2\pi f)^2 m = (2\pi \times 5)^2 \times 0.5 = 492.98 \text{ N/m}$$

To find the damping constant

Quality factor is given by

$$Q = \frac{\omega_0}{\gamma}$$

$$\gamma = \frac{\omega_0}{Q} = \frac{2\pi f}{Q} = \frac{2\pi \times 5}{75} = 0.4186$$

$$\gamma m = b = 0.4186 \times 0.5 = 0.209 \text{ Kg/sec}$$

11. A vibrating system of natural frequency 500 cycles/sec is forced to vibrate with a periodic force/unit mass of amplitude 100×10^{-5} N/Kg in the presence of a damping/unit of 0.01×10^{-3} rad/sec. calculate the maximum amplitude of vibration of the system.

SOLUTION:

$f=500$ cycles/sec, $F_0 = \frac{F}{m} = 100 \times 10^{-5}$ N/Kg, $b = 0.01 \times 10^{-3}$ rad/sec

$$\omega = 2\pi f = 2\pi \times 500 = 3140 \text{ rad/sec}$$

For forced oscillation, maximum amplitude

$$A_{max} = \frac{F_0}{b\omega_0} = \frac{100 \times 10^{-5}}{0.01 \times 10^{-3} \times 3140} = 0.0318 \text{ m}$$

12. A body of mass 500 gm is attached to a spring and the system is driven by an external periodic force of amplitude 15 N, and frequency 0.796 Hz. The spring extended by a length of 88mm under the given load. Calculate the amplitude of oscillation if the resistance coefficient of the medium is 5.05 Kg/s. Ignore the mass of the spring.

SOLUTION:

$m=500 \text{ gm}=0.5 \text{ Kg}$, $F=15 \text{ N}$, $f=0.796 \text{ Hz}$, $x=88 \times 10^{-3} \text{ m}$, $b=5.05 \text{ kg/s}$.

Angular frequency of the applied force,

$$\omega = 2\pi f = 2\pi \times 0.796 = 4.998 \text{ rad/sec}$$

Force constant,

$$k = \frac{F}{x} = \frac{mg}{x} = \frac{0.5 \times 9.8}{88 \times 10^{-3}} = 55.68 \text{ N/m}$$

Natural frequency of oscillation of the body,

$$\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{55.68}{0.5}} = 10.55 \text{ rad/sec}$$

Damping factor,

$$\gamma = \frac{b}{m} = \frac{5.05}{0.5} = 10.1 \text{ rad/sec}$$

The amplitude of oscillation is given by,

$$A = \frac{\frac{F_0}{m}}{\sqrt{(\omega_0^2 - \omega^2)^2 + (\gamma\omega)^2}}$$

$$= \frac{\frac{15}{0.5}}{\sqrt{(10.55^2 - 4.998^2)^2 + (10.1 \times 4.998)^2}} = \frac{30}{\sqrt{7451.57 + 2548.21}} = \frac{30}{99.99}$$

$$= 0.3m$$

13. Calculate the peak amplitude of vibration of a system whose natural frequency is 1000 Hz when it oscillates in a resistive medium for which the value of damping/unit mass is 0.008 rad/sec under the action of an external periodic force/unit mass of amplitude 5 N/Kg, with tunable frequency.

SOLUTION:

$f=1000 \text{ Hz}$, $\gamma = 0.008 \text{ rad/sec}$, $\frac{F_0}{m} = 5 \text{ N/Kg}$

Angular frequency of the applied force,

$$\omega = 2\pi f = 2\pi \times 1000 = 6280 \text{ rad/sec}$$

The amplitude of oscillation is given by,

$$A = \frac{\frac{F_0}{m}}{\sqrt{(\omega_0^2 - \omega^2)^2 + (\gamma\omega)^2}}$$

For maximum amplitude is achieved when the frequency ω of the applied force is tuned to match the natural frequency ω_0 of the system.

$\omega = \omega_0$

$$A_{max} = \frac{\frac{F_0}{m}}{\sqrt{(\omega^2 - \omega^2)^2 + (\gamma\omega)^2}} = \frac{\frac{F_0}{m}}{\sqrt{0^2 + (\gamma\omega)^2}} = \frac{\frac{F_0}{m}}{\sqrt{(\gamma\omega)^2}} = \frac{F_0}{\gamma\omega}$$

$$A_{max} = \frac{5}{0.008 \times 6280} = 0.099m$$

14. A vibrating system of natural frequency 500 cycles/sec., is forced to vibrate with a periodic force/unit mass of amplitude 100×10^{-5} N/Kg in the presence of a damping/unit mass of 0.01×10^{-3} rad/sec. calculate the maximum amplitude of vibration of the system.

SOLUTION:

$$f=500 \text{ cycles/sec, } \frac{F_0}{m} = 100 \times 10^{-5} \text{ N/Kg, } \gamma = 0.01 \times 10^{-3} \text{ rad/sec}$$

Angular frequency of the applied force,

$$\omega = 2\pi f = 2\pi \times 500 = 3140 \text{ rad/sec}$$

The maximum amplitude of oscillation is given by

$$A_{max} = \frac{\frac{F_0}{m}}{\gamma\omega} = \frac{100 \times 10^{-5}}{0.01 \times 10^{-3} \times 3140} = 0.0318 \text{ m}$$

15. A mass of 2 Kg suspended by a spring of force constant 51.26 N/m is executing damping simple harmonic oscillations with a damping of 5 Kg/ sec. Identify whether it is the case of underdamping or of overdamping. Also estimate the value of damping required for the oscillations to be critically damped (ignore the mass of the spring).

SOLUTION: $m=2$ Kg, $k=51.26$ N/m, $b=5$ Kg/ sec

Damping factor, $b^2=(5)^2=25$

Let us calculate, $4mk=4 \times 2 \times 51.26=410.08$

Therefore, $b^2 < 4mk$ which is under damping

For critically damped oscillation, $b^2 = 4mk$

$$b^2 = 4mk = 4 \times 2 \times 51.26 = 410.08$$

$$b = \sqrt{410.08} = 20.25 \text{ kg/sec}$$

16. A 20gm oscillator with natural angular frequency 10 rad/ sec is vibrating in damping medium. The damping factor is proportional to the vibrator. If the damping coefficient is 0.17, how does the oscillation decay?

SOLUTION: $m=20\text{gm}=0.02$ Kg, $\omega_0=10$ rad/ sec, $b=0.17$ Kg/sec

Natural frequency of oscillation of the body,

$$\omega_0 = \sqrt{\frac{k}{m}} \Rightarrow \omega_0^2 m^2 = k$$

Force constant,

$$k = 10^2 \times 0.02^2 = 0.04 \text{ N/m}$$

Damping factor

$$b^2 = 0.17^2 = 0.0289 \text{ --- (1)}$$

$$\text{let us consider, } 4mk = 4 \times 0.02 \times 0.04 = 3.2 \times 10^{-3} \text{ --- (2)}$$

comparing 1 and 2, $b^2 < 4mk$

1.3 SHOCK WAVES

1.3.1 Introduction

The true history of shock wave research in this period bears little direct relation to the problems seen in Explosive Devices, Metal Cutting and Shaping with Explosives, Jetting and Penetration, Wave Shaping, Explosive Welding, Metallurgy: Twinning and Transformation Bands, Phase Transformation in Steels Powder Compaction, Synthesis of Diamond, and Cracking of Tar Sand etc... It was a time of simplification and of a rebuilding from fundamentals of physics. The modern shock wave research was published by J.M Walsh and R. H. Christian in March, 1955. Shocks were generated by plane wave explosive incidents. In which pressure was considered to be most significant factor in condensed matter physics. Static high pressure research was observed and there was a considerable amount of effects in producing the shock waves.

1.3.2 Acoustic waves: These are also known as sound waves, a type of longitudinal waves that propagate by means of adiabatic compression and rarefactions. This will have the same direction of vibration as their direction of travel. These sound waves are audible has the frequencies lie between 20 Hz to 20,000 Hz. Travel at a speed of 343 ms^{-1} at STP.

The longitudinal waves whose frequencies less than 20 Hz called “**infrasonic waves**”. These waves are not sensitive to our ear.

The longitudinal waves whose frequencies greater than 20,000 Hz called “**Ultrasonic waves**”. These waves are not sensitive to our ear.

1.3.3 Shock Waves: Shock waves are strongly pressure disturbances that are generated suddenly either by explosion or by a plane flying at a supersonic speed.

1.3.4 Mach number: it is defined as the ratio of the speed of object in medium to the speed of the sound in that medium.

$$\text{Mach number} = \frac{\text{Speed of object in medium}}{\text{Speed of sound in that medium}}$$

$$M = \frac{v_o}{v_s}$$

1.3.3 Subsonic waves: These are mechanical wave’s moves with the speed less than the speed of sound waves in fluid called subsonic waves. Therefore the Mach number for such waves is less than unity i.e., $M < 1$. Most of the speed of vehicle moves with less than the speed of sound.

1.3.4 Transonic waves: The mechanical wave's move with the speed is equal to the speed of sound waves in fluid called subsonic waves. Therefore the Mach number for such waves is less than unity i.e., $M = 1$.

1.3.5 Supersonic Waves: The mechanical waves which move with the speed greater than the speed of sound waves are called supersonic waves. The Mach number for such waves is greater than unity and ranges from 1 to 3 i.e., $1 < M < 3$.

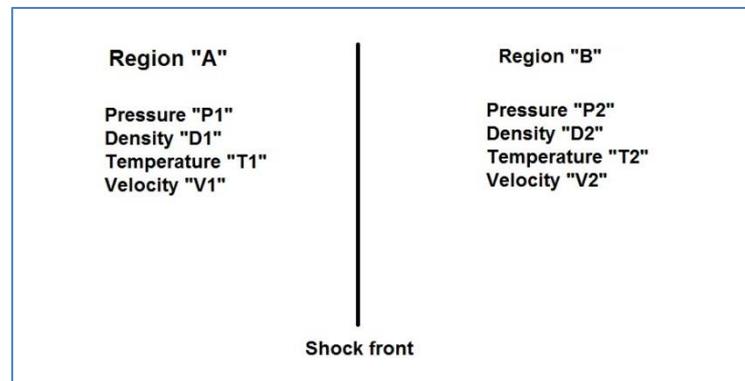
1.3.6 Characteristic properties of Shock waves

- i. Shock waves travel in a medium with Mach number $(M) > 1$.
- ii. Shock waves obey the laws of fluid dynamics.
- iii. When shock waves pass through a medium the entropy of the system increases.
- iv. When shock waves pass through a medium the changes are adiabatic.
- v. General wave properties cannot be associated with shock waves.

1.3.7 Basics of conservation of Mass, Energy and Momentum

i. Control volume: For an inertial frame of reference it is a volume fixed in space or moving with constant velocity through which the fluid flows. The surface enclosing the control volume is referred to as the control surface.

Consider a rectangular region of fluid (say air) around a normal shock wave as shown in the figure. Normal shock waves are the basic type of shock waves which are in a direction perpendicular to the flow. The left of the shock wave we have the initial conditions and final conditions to the right. M_1 , V_1 , p_1 , T_1 and ρ_1 represent the initial conditions of the flow and M_2 , V_2 , p_2 , T_2 and ρ_2 represent the final conditions of the flow.



The following are the assumptions with regard to the flow

- a. It is a steady flow.
- b. It is an adiabatic system.
- c. There are no viscous forces acting between the system and boundaries.
- d. There are no external forces acting.

ii. Conservation of Mass

The basic law of conservation is mass can neither be created nor destroyed. For a closed system the total mass remains constant. Mathematically

$$m_{\text{sys}} = \text{Constant}$$

Hence

$$\frac{dm_{\text{sys}}}{dt} = 0$$

For a control volume or open system conservation of mass is expressed in the rate form

as
$$\dot{m}_{\text{in}} - \dot{m}_{\text{out}} = \frac{dm_{\text{sys}}}{dt}$$

Here \dot{m}_{in} and \dot{m}_{out} are the mass flow into and out of the control volume respectively. $\frac{dm_{sys}}{dt}$ is the rate of change of mass within the control volume boundaries. By applying the law of conservation of mass to the system, we get

$$\rho_1 V_1 = \rho_2 V_2$$

iii. Conservation of Momentum

It is given by the total momentum of the system is always conserved. By applying Newton's second law of motion to the control volume the conservation of momentum is "any change in momentum of the fluid (air) within a control volume be due to the net flow of fluid (air) into the volume and the action of external forces on the fluid within the volume."

$$P_1 + \rho_1 v_1^2 = P_2 + \rho_2 v_2^2$$

iv. Conservation of energy

It deals with energy can neither be created nor destroyed. In other words total energy of a system remains constant. In case of a fluid system under adiabatic conditions the total energy entering the system must be same as the total energy leaving the system. The total energy of the control volume remains constant.

$$h_1 + \frac{v_1^2}{2} = h_2 + \frac{v_2^2}{2}$$

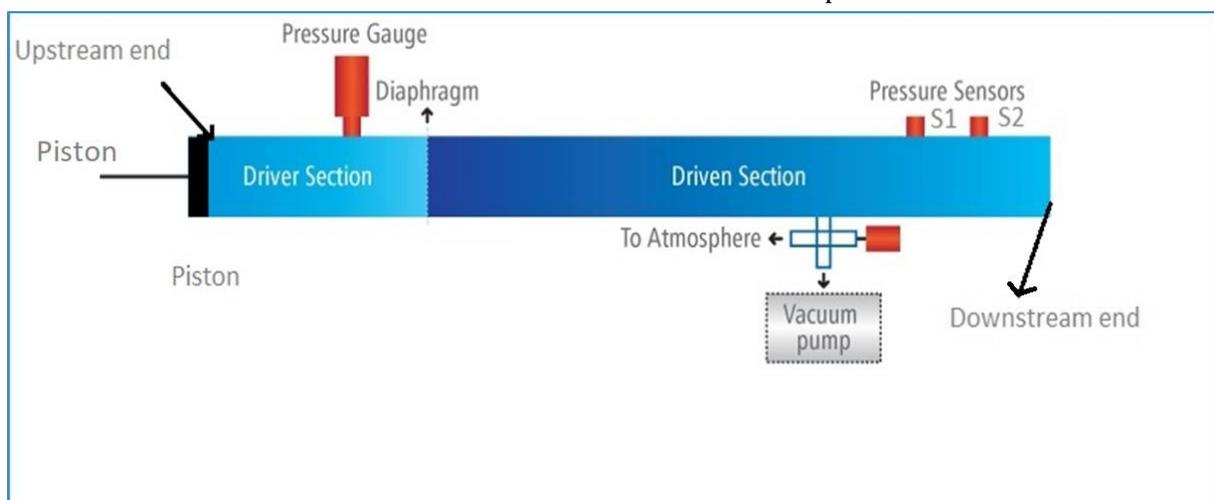
1.3.8 Hand operated Reddy Shock tube

"It's a hand operated tube capable of generating Shock Waves in a small region. With the Mach number of waves more than 1.5"

Principle: It operates on the principle of **Free Piston-drive Shock Tube**.

Construction: The setup consists of long cylindrical stainless tube of length one meter and diameter 30mm. The tube is divided into two sections.

- One is called the driver section which maintains at high pressure
- Another called driven section which maintains at low pressure.



A diaphragm made of aluminium or plastic of thickness separates the two sections. A piston is fitted at one free end of driver section for manual operation. Other end of driven section is

closed. A pressure gauge is fixed in driver section near diaphragm and two pressure sensors are fixed at closed end of the driven section with separation of 70mm. The driver section is filled with driver gas and gas in driven section is called driven gas.

Working: The driver section initially kept at elevated pressure and the driven section at low pressure. When the piston is pushed hard with high speed, the gas gets compressed producing the heat by adiabatic compression. The diaphragm bursts due to high pressure heated gas creating a shock wave in gas of driven section.

This shock wave instantaneously raises the pressure and temperature in driven gas. This shock wave will reflect from downstream end. This undergoes further compression and increase in the value of pressure and temperature. This gas will be reflected back and forth between two ends till compressions reduce. The pressure values can be read from sensors S_1 and S_2 . This kind of shock waves has many applications.

Characteristics of Shock tube

- It operates on the principle of **Free Piston-drive Shock Tube (FPST)**.
- It can generate the shock waves with Mach number greater than 1.5
- The produced shock waves will have the pressure about more than the few hundred atmospheric pressure and temperature greater than 10000K. Hence, these shock waves can be used for material processing and synthesis of materials.
- The shock waves produced in smaller diameter tubes can be used for studying the blast induced traumatic brain injuries.
- It can be used to generate shock waves with Mach number greater than 2 by proper dimensional analysis.

1.3.9 Applications of Shock Waves

The following are the applications of the shock waves.

- Mach reflection of a shock wave is used to remove micron size dust particles from the surface of silicon wafers.
- Shock waves are used in medical therapy in orthopaedics and for breaking kidney stones.
- Shock waves are used in pencil industries to impregnate preservatives into wood slats.
- Shock waves are used in sandal oil extraction.
- Shock waves are used in biological cell transformation.
- Shock waves are used in geophysics.

Numerical problems:

- A body is moving with a speed of 3000 Km/hr. classify the speed of the body based on Mach number, given velocity of sound in air is 343 m/s and also calculate the Mach angle.

SOLUTION:

$$v = 3000 \frac{Km}{hr} = \frac{3000 \times 1000 \text{ m}}{3600 \text{ s}} = 833.33 \text{ m/s}$$

$$v_{\text{sound}} = 343 \text{ m/s}$$

Mach number,

$$M = \frac{v}{v_{\text{sound}}} = \frac{833.33}{343} = 2.4295$$

$$\text{mach angle } \theta = \sin^{-1}\left(\frac{1}{M}\right) = \sin^{-1}\left(\frac{1}{2.4295}\right) = 0.4242''$$

2. The difference between two pressure sensors in a shock tube is 150 mm. The time taken by shock waves to travel this distance is 0.3 m sec. if the velocity of sound under the same condition is 343 m/s, find the Mach number of the shock wave.

SOLUTION: distance between sensor= $d=150 \text{ mm}=150 \times 10^{-3} \text{ m}$,

Time taken distance wave to travel, $t=0.3 \text{ m sec}=0.3 \times 10^{-3} \text{ sec}$, $v_{\text{sound}}=343 \text{ m/s}$

Mach number,

$$M = \frac{v}{v_{\text{soundd}}}$$

$$\text{velocity of the shock waves, } v = \frac{d}{t} = \frac{150 \times 10^{-3}}{0.3 \times 10^{-3}} = 500 \text{ m/s}$$

$$M = \frac{v}{v_{\text{soundd}}} = \frac{500}{343} = 1.4577$$

3. In a Shock tube experiment, it was found that the time taken to travel between the two sensors is 195 μ sec. if the distance between the two sensors is 100 mm, find the Mach angle. Given velocity of sound=343 m/s

SOLUTION: distance between sensor= $d=100 \text{ mm}=100 \times 10^{-3} \text{ m}$,

$t=195 \mu \text{ sec}=195 \times 10^{-6} \text{ sec}$, $v_{\text{sound}}=343 \text{ m/s}$

$$\text{velocity of the shock waves, } v = \frac{d}{t} = \frac{100 \times 10^{-3}}{195 \times 10^{-6}} = 512.82 \text{ m/s}$$

$$M = \frac{v}{v_{\text{soundd}}} = \frac{512.82}{343} = 1.4951$$

$$\text{mach angle } \theta = \sin^{-1}\left(\frac{1}{M}\right) = \sin^{-1}\left(\frac{1}{1.4951}\right) = 0.7326''$$

Reference:

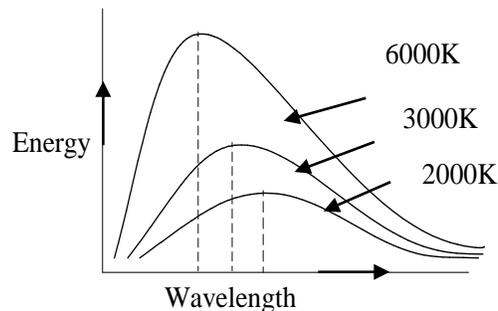
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MODULE - 1

Modern Physics and Quantum Mechanics

Blackbody Radiation spectrum:

A Blackbody is one which absorbs the entire radiation incident on it and emits all the absorbed radiation when it is hotter. A true blackbody does not exist practically. A blackbody designed by Wein has features very close to the true blackbody. A blackbody at a particular temperature found to emit a radiation of all possible wavelengths. It is a continuous spectrum starting from certain minimum wavelength to maximum wavelength. The maximum intensity (E_{\max}) corresponds to a particular wavelength (λ_m). For different temperatures of the black body, there are different curves. As the temperature of the body increases, the wavelength corresponding to maximum intensity shifts towards lower wavelength side. The distribution of energy in black body radiation is shown in the following fig.



Wein's, Rayleigh-Jeans and Planck have given their explanations to account these observed experimental facts as follows:

1. Wein's Displacement Law:

The law states that "the wavelength of maximum intensity is inversely proportional to the absolute temperature of the emitting body, because of which the peaks of the energy curves for different temperatures get displaced towards the lower wavelength side".

$$\text{i.e. } \lambda_m \propto \left(\frac{1}{T}\right) \text{ or } \lambda_m T = \text{constant} = 2.898 \times 10^{-3} \text{ mK}$$

Wein showed that the maximum energy of the peak emission is directly proportional to the fifth power of absolute temperature.

$$E_m \propto T^5 \quad \text{or} \quad E_m = \text{constant} \times T^5$$

Wein's Energy Distribution law: The relation between the wavelength of emission and the temperature of the source is

$$U_\lambda d\lambda = C_1 \lambda^{-5} e^{-\left(\frac{C_2}{\lambda T}\right)} d\lambda$$

Where $U_\lambda d\lambda$ is the energy / unit volume in the range of wavelength λ and $\lambda+d\lambda$, C_1 and C_2 are constants.

This is called Wein's law of energy distribution in the black body radiation spectrum.

Drawbacks of Wein's law:

Wein's law holds good for the shorter wavelength region and high temperature of the source. It failed to explain gradual drop in intensity of radiation corresponding to longer wavelength greater than the peak value.

2. Rayleigh-Jeans Law:

Rayleigh derived an equation for the blackbody radiation based on principle of equipartition of energy. The principle of equipartition of energy suggests that an average energy kT is assigned to each mode of vibration. The number of vibrations/unit volume whose wavelength is in the range of λ and $\lambda+d\lambda$ is given by $8\pi\lambda^{-4}d\lambda$.

The energy/unit volume in the wavelength range λ and $\lambda + d\lambda$ is

$$U_{\lambda}d\lambda = 8\pi\lambda^{-4}kTd\lambda$$

Where k is Boltzmann constant = $1.38 \times 10^{-23} \text{ J/K}$.

This is Rayleigh-Jean's equation. According to the above equation the energy radiated by the blackbody decreases with increasing wavelength.

Drawbacks of Rayleigh-Jeans Law: (or Ultra Violet Catastrophe)

Rayleigh-Jeans Law predicts to radiate all the energy at shorter wavelength side but it does not happen so. A black body radiates mainly in the infra-red or visible region of electromagnetic radiation spectrum and intensity of radiation decreases down steeply for shorter wavelengths. Thus, the Rayleigh-Jeans Law fails to explain the lower wavelength side of the spectrum. This is referred to as ultra-violet Catastrophe.

3. Planck's Law:

Planck assumed that walls of the experimental blackbody consists larger number of electrical oscillators. Each oscillator vibrates with its own frequency.

- i) Each oscillator has an energy given by integral multiple of $h\nu$ where h is Planck's constant & ν is the frequency of vibration.
 $E = nh\nu$ where $n = 1, 2, 3 \dots$ etc.
- ii) An oscillator may lose or gain energy by emitting or absorbing radiations of frequency ν where $\nu = \Delta E/h$, ΔE is difference in energies of the oscillator before and after the emission or absorption take place.

Planck derived the law which holds good for the entire spectrum of the blackbody radiation as

$$U_{\lambda}d\lambda = \frac{8\pi hc}{\lambda^5} \left[\frac{1}{e^{h\nu/kt} - 1} \right] d\lambda \quad (\text{since } \nu = c/\lambda) \quad \longrightarrow \quad (1)$$

This is Planck's Radiation Law.

Reduction of Planck's law to Wein's law and Rayleigh Jeans law:

1) For shorter wavelengths, $\nu = c/\lambda$ is large.

When ν is large, $e^{h\nu/kT}$ is very large.

$$\begin{aligned} \therefore e^{h\nu/kT} &\gg 1 \\ \therefore (e^{h\nu/kT} - 1) &\approx e^{h\nu/kT} = e^{hc/\lambda kT} \end{aligned}$$

Substituting in eqn 1:

$$U_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \left[\frac{1}{e^{hc/\lambda kT}} \right] d\lambda = C_1 \lambda^{-5} e^{\left(\frac{-C_2}{\lambda T}\right)} d\lambda$$

Where $C_1 = 8\pi hc$ and $C_2 = hc/k$

This is the Wein's law of radiation.

2) For longer wavelengths $\nu = c/\lambda$ is small.

When ν is small $h\nu/kT$ is very small.

Expanding $e^{h\nu/kT}$ as power series:

$$\begin{aligned} e^{h\nu/kT} &= 1 + h\nu/kT + (h\nu/kT)^2 + \dots \\ &\approx 1 + h\nu/kT. \end{aligned}$$

\therefore If $h\nu/kT$ is small, its higher powers are neglected.

$$\therefore e^{h\nu/kT} - 1 = \frac{h\nu}{kT} = \frac{hc}{\lambda kT}$$

Substituting in eqn 1:

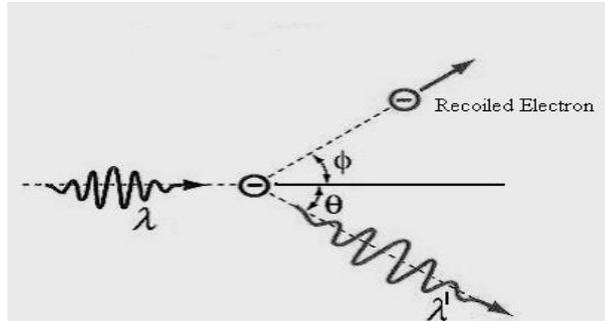
$$\begin{aligned} U_\lambda d\lambda &= \left[\frac{8\pi hc}{\lambda^5 \left(\frac{hc}{\lambda kT}\right)} \right] d\lambda \\ &= \left[\frac{8\pi kT}{\lambda^4} \right] d\lambda \end{aligned}$$

This is Rayleigh Jeans Law of Radiation.

Compton Effect:

The scattering of a photon by an electron is called as Compton effect or Compton scattering.

When a photon of wavelength ' λ ' is scattered by an electron in the direction making an angle ' θ ' with the direction of incidence, the wavelength of the scattered photon increases. Its wavelength is λ' . The electron recoils in a direction making an angle ' ϕ ' with the incident direction of photon. The difference in the wavelength ($\lambda' - \lambda$) is called the Compton shift. Compton found that λ' is independent of the target material, but depends on the angle of scattering.



If λ is the wavelength of the incident photon, its energy E is given by $E=hc/\lambda$ where 'h' is the Planck's constant, 'c' is the velocity of light, ' λ ' is the wavelength of the incident photon. If λ' is the wavelength of the scattered photon, its energy E' is given by

$$E' = hc/\lambda'$$

The energy of the scattered photon is reduced from E to E' . The difference of energy is carried by recoiling electron at an angle ' ϕ ' with the incident direction of photon.

Applying the laws of conservation of energy and conservation of momentum Compton obtained an expression for change in wavelength $\Delta\lambda$ given by
$$\Delta\lambda = \lambda' - \lambda = \frac{h}{mc}(1 - \cos\theta)$$

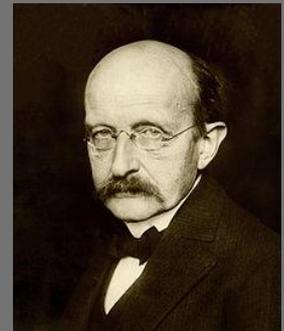
Where 'm' is the mass of the electron, h/mc is called as Compton wavelength.

Compton Effect explains particle nature of light.

Quantum Mechanics

The behavior of entities at atomic scale more pronounced

The gestation of Quantum Physics has been very long and its phenomenological foundations were various. Historically the original idea came from the analysis of the black body spectrum. This is not surprising since the blackbody, in fact an oven in thermal equilibrium with the electromagnetic radiation, is a simple and fundamental system once the laws of electrodynamics are established. As a matter of fact many properties of the spectrum can be deduced starting from the general laws of electrodynamics and thermodynamics; the crisis came from the violation of energy equipartition. This suggested to Planck the idea of quantum, from which everything originated.



Max Planck

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VTU Syllabus: Introduction to Quantum mechanics, Wave nature of particles, Heisenberg's uncertainty principle and applications (Non-confinement of electron in the nucleus), Schrodinger time independent wave equation, Significance of Wave function, Normalization, Particle in a box, Energy Eigen values of a particle in a box and probability densities.

Introduction to Quantum mechanics: The discoveries and insights over the last three centuries share a characteristic feature: seemingly unconnected phenomena turned out to be manifestations of the same fundamental principle. It was a period of unification of disparate fields of experience. Here are some of the most important steps. Newton showed that the motion of the planets is governed by the same law as the free fall of an object on earth. Thus, the classical theory unifies terrestrial and celestial mechanics. In contrast to the belief of the ancients, the classical theory shows that the world of the earth and the macroscopic phenomenon.

For a long time, the phenomena of electricity, magnetism, and light appeared to be unconnected. In the first half of the nineteenth century, one of the great unifications of physics took place. Faraday and Maxwell, together with many others, were able to show that all three phenomena are manifestations of the electromagnetic field. And so the field concept entered into physics. The simplest example is the electric field of an electric charge that exerts a force on another charge when the latter falls within its range. An electric current produces a magnetic field that exerts a force on magnetic materials. In Quantum mechanics the fundamental concepts were not too different from those of our everyday experience, such as particle, position, speed, mass, force, energy, and even field. We often refer to those concepts as classical. The world of atoms cannot be described and understood with those concepts. For atoms and molecules, the ideas and concepts formed in dealing with the objects in our immediate environment no longer suffice. One needs a new concept to understand the properties of atoms. The quantum mechanics changed our old concepts of reality in many respects.

According to Max Planck the energy could be taken only certain discrete values as follows

- i. A System can absorb and emit the radiations in discrete packets called quanta.
- ii. If ' ν ' is the frequency of an electromagnetic oscillator. Then its energy is proportional to ν .

$$\text{i.e., } E = h\nu, 2h\nu, 3h\nu, \dots$$

Where h is Planck's constant and its value is 6.625×10^{-34} JS.

In other words it states that exchange of energy between the radiation and matter cannot take place continuously. The energies of the atoms are said to be **quantized** and the allowed energy levels are called as **quantum levels**.

$$E = nh\nu$$

A Black body which absorbs all radiations and emits all radiations irrespective of wavelengths can be explained only based on quantum mechanical principles. Then, the energy density corresponds to the emitted radiation in the wavelength range of λ and $\lambda + d\lambda$ is given by

$$E_{\lambda} d\lambda = \frac{8\pi hc}{\lambda^2} \frac{d\lambda}{\left[e^{\frac{hc}{\lambda kT}} - 1 \right]}$$

Wave nature of particles: The light, believed to be a wave, exhibited particle properties and that electrons, believed to be particles, exhibited wave properties. Let us call it the wave-particle duality. Louis de-Broglie extended the wave particle duality of light to all the fundamental entities of physics such as electrons, protons neutrons etc. de-Broglie put a bold suggestion that, if radiation can behave as particle under certain circumstances then, one can even expect that entities which ordinarily behaves as particles to exhibit wave properties under appropriate circumstances.

The hypothesis of de-Broglie was that the dual nature that is wave –particle behavior of radiation applies equally well to matter. Just as a photon has a light wave associated with it, governs its motion. Such a wave associated with matter are called matter waves or de-Broglie waves and the hypothesis is called de-Broglie hypothesis. de-Broglie hypothesis opened up a new thinking in almost all the fields of Physics. In fact it can be treated as the new beginning of the Modern Physics.

The expression of the wavelength associated with a material particle can be derived on the analogy of radiation as follows:

Considering the Plank’s theory of radiation, the energy of a photon (quantum) is given by

$$E = h\nu \text{ and } E = mc^2$$

Equating above equations, we get

$$h\nu = mc^2$$

$$\frac{hc}{\lambda} = mc^2$$

$$\lambda = \frac{h}{mc}$$

Where c is the speed of light and $h = 6.62 \times 10^{-34}$ JS is Planck’s constant.

The momentum of a particle of mass ‘ m ’ and velocity ‘ v ’ is $p = mv$ and its de-Broglie wavelength is accordingly

$$\lambda = \frac{h}{mv}$$

This physically means that a material particle of mass ‘ m ’ moving with the velocity ‘ v ’ has a wave associated with it of de-Broglie wavelength.

This physically means that a material particle of mass ‘ m ’ moving with the velocity ‘ v ’ has a wave associated with it of de-Broglie wavelength.

If ‘ E ’ is the kinetic energy of the particle, then De-Broglie wavelength is $\lambda = \frac{h}{\sqrt{2mE}}$

For an accelerated charged particle at a potential difference of ‘ V ’, the de-Broglie’s wavelength is

$$\lambda = \frac{h}{\sqrt{2meV}}$$



Heisenberg's Uncertainty Principle: To regard a moving particle as a wave group implies that there are fundamental limits to the accuracy with which we can measure such "particle" properties as position and momentum. According to Heisenberg's uncertainty principle: It is impossible to know both the exact position and momentum of a particle at the same time, It states that "The product of inherent uncertainties involved in the measurements position and momentum simultaneously is greater than or equal to greater than $\frac{h}{4\pi}$ ".

$$\Delta x \cdot \Delta p \geq \frac{h}{4\pi}$$

Another forms of the uncertainty principle concerns energy and time is given by

$$\Delta E \cdot \Delta t \geq \frac{h}{4\pi}$$

The Uncertainty principle between angular displacement and angular momentum is given by

$$\Delta J \cdot \Delta \theta \geq \frac{h}{4\pi}$$

Significance of Uncertainty principle: The Physical Significance of this principle is that one should not think of exact position or an accurate value of momentum of the particle. Instead one should think of probability of finding the particle in a certain region or the probability of finding the momentum of the particle. The estimation of such probabilities is made by means of certain mathematical function called wavefunction in quantum mechanics.

Applications of uncertainty principle:

Non-existence of electrons in atomic nucleus:

We know that, the diameter of nucleus cannot exceed $2 \times 10^{-14} m$. Let us assume that, if an electron exists inside the nucleus. This means, Uncertainty in position cannot exceed the diameter of the nucleus. Hence $\Delta x \leq 2 \times 10^{-14} m$.

We know from Heisenberg uncertainty principle

$$\Delta x \cdot \Delta p \geq \frac{h}{4\pi}$$

$$\Delta p \geq \frac{6.626 \times 10^{-34}}{4 \times 3.14 \times 2 \times 10^{-14}}$$

$$\Delta p \geq 5.2754 \times 10^{-21} \text{ kgms}^{-1}$$

Therefore, the uncertainty in momentum of the electron is $5.2754 \times 10^{-21} \text{ kgms}^{-1}$ and hence, the momentum of an electron is equal to uncertainty in its momentum.

$$p \geq 5.2754 \times 10^{-21} \text{ kgms}^{-1}$$

Then the relativistic energy of the electron in the nucleus is given by

$$E = \frac{p^2}{2m}$$

Where m is the rest mass of the electron and p is the momentum

$$E = \frac{[5.2754 \times 10^{-21}]^2}{[2 \times 9.1 \times 10^{-31}]}$$

$$E = 1.5291 \times 10^{-11} \text{ J}$$

or $E = 95.572 \text{ MeV}$

The value indicates that an electron requires energy of 95.572 MeV to exist inside the nucleus. But

the measurements made in the beta decay shows that the kinetic energy of the order of 3 – 4 MeV. The actual energy of the electrons exists in various orbits outside the nucleus is less than the value 95.572 MeV, if it exists inside the nucleus. Therefore, it is clear the no electron can exists inside the nucleus.

Wave function:

We know that matter exhibits wave like behavior under certain conditions. When the momentum of the particle is well defined, the wave can be of infinite extent. Therefore a particle moving along x-axis with well defined momentum is described by an infinite plane wave $\psi(x, t)$ and is given by

$$\psi(x, t) = A e^{i(kx - \omega t)}$$

It is a function of space variables (x,y,z) and time 't'

Where ω angular frequency and k is wave vector

In the case of electromagnetic waves, the electromagnetic waves vary with space and time. In case of sound waves, it is describes pressure variation in space and time. In other words to describe the wave function one requires quantity which varies in space and time. In analogy with these, to describe the matter waves associated with the particle motion, one requires a quantity which varies with space and time, this variable quantity is called as wave function $\psi(x, t)$. The solution describes wave aspect, particle aspect in consistent with the uncertainty principle.

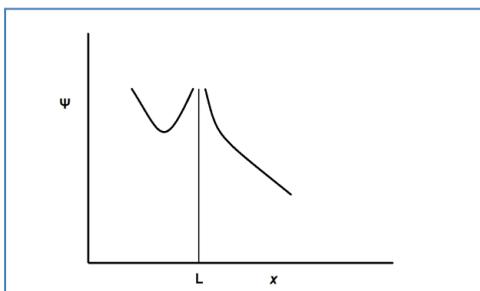
Physical significance of wave function:

The wave function $\psi(x, t)$ signifies the probability of finding the particle described by the wave function at the point $\vec{r} = (x, y, z)$ and at time 't'

Properties of wave function:

The knowledge of the wave function of the particles can be evaluated by knowing the values of $\psi(x, t)$ from the Schrödinger wave equation.

- i) The wave function $\psi(x, t)$ is single-valued everywhere:** The wave function should have single value rather than multiple values. Consider a wave function $\psi(x, t)$ has an interval of wave function ψ_1, ψ_2 and ψ_3 .

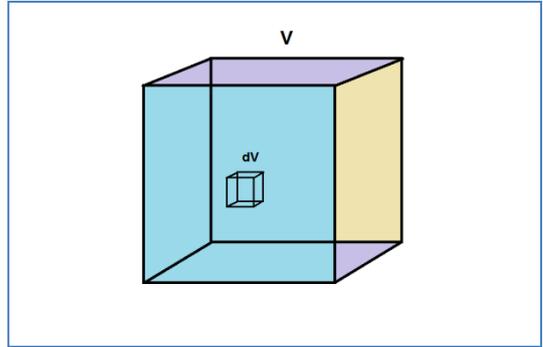


- ii) Wave function is finite everywhere:** Consider a wave function ψ as a function of x as shown in the figure 1.4. The wave function is infinite at $x = L$, Which means the probability of finding the particle is infinitely large at $x = L$. This violates the Heisenberg uncertainty principle. Thus the wave function becomes unacceptable. Therefore, the wave function must have a finite value everywhere.

iii) Wave function and its derivatives are continues everywhere: When the wave function is continuous and finite everywhere, the first and second derivatives are also continuous which is essential to evaluate the Schrödinger equation. Hence wave functions must be continuous everywhere.

- iv)** It is large in magnitude where the particle is likely to be located and small in other places.
- v)** The wave function describes the behavior of a single particle not the statistical distribution of a number of such quanta.
- vi)** It can interfere with itself (phenomenon of electron diffraction)

Probability density: Consider a particle whose position is independent of time in a volume V inside which particle is known to be present in a volume element dV . Let ψ be the wave function associated with the particle, and then the square of the wave function associated with the particle is interpreted as measure of probability density.



That is $|\psi|^2$ is the probability density per volume element dV , that the particle will be found in that volume element.

$$P(x, t) = |\psi|^2 dV$$

If the wave function is complex, then, the probability density is product of wave functions ψ and its complex conjugate ψ^* , therefore the probability density is given by

$$P(x, t) = \psi \psi^*$$

Normalization: If the particle exists somewhere at all the time. Let ψ be the wave function associated with the particle, then the probability density of particle in a volume element is $|\psi|^2 dV$. If we further extend the case where the particle is definitely present in some region or space, As per the statistical rule the normalization is given by.

$$\int_0^V |\psi|^2 dV = 1$$

This is based on the fact that, a value 1 for probability means, it is clearly a certainty. However, in the above case if we are not at all certain about locating the particle in a finite volume anywhere in the space, then the expectation will become limited to the extent that it exists somewhere in space. Then the limits in equation extend to $-\infty$ to $+\infty$ and the probability becomes.

$$\int_{-\infty}^{+\infty} |\psi|^2 dV = 1$$

This process is called Normalization.

Schrodinger's time independent wave equation:

The wave function of matter wave which is confined to a small region of space is given by

$$\psi(x, t) = A e^{i(kx - \omega t)} \quad (1)$$

In many cases the potential energy does not depend on time ' t ' explicitly. In these cases, there is no external force acts on the particle. Hence, the potential energy depends on the position of the particle; the wave function for such cases can be obtained as follows.

Differentiating the equation (1) twice with respect to ' x ' we get

$$\frac{d^2\psi}{dx^2} = -k^2\psi \quad (2)$$

Differentiating the equation (1) twice with respect to ' t ' we get

$$\frac{d^2\psi}{dt^2} = -\omega^2\psi \quad (3)$$



We have Phase velocity $v_{phase} = \frac{\omega}{k}$ or $k = \frac{\omega}{v_{phase}}$ (4)

Substituting equation (4) in (2), we get

$$\frac{d^2\psi}{dx^2} = -\frac{\omega^2}{v_{phase}^2}\psi \quad (5)$$

Using (3) in (5),
$$\frac{d^2\psi}{dx^2} = \frac{1}{v_{phase}^2} \frac{d^2\psi}{dt^2}$$

This is the equation for the travelling wave. Now, we have $k = \frac{2\pi}{\lambda}$ (6)

Substituting equation (6) in (2), we get

$$\frac{d^2\psi}{dx^2} = -\frac{4\pi^2}{\lambda^2}\psi \quad (7)$$

The total energy of the particle is

$$E = KE + PE \quad \text{or}$$

$$E = \frac{p^2}{2m} + V_x \quad (8)$$

Where, V_x is the potential energy, it depends only on position and is independent of time.

The De-Broglie's wave length is $\lambda = \frac{h}{p}$ or $P = \frac{h}{\lambda}$ (9)

Substituting equation (9) in (8), we get

$$E - V_x = \frac{h^2}{2m\lambda^2}$$

Or
$$\frac{2m(E - V_x)}{h^2} = \frac{1}{\lambda^2} \quad (10)$$

Substituting equation (10) in (7), we get

$$\frac{d^2\psi}{dx^2} = -\frac{8\pi^2m(E - V_x)}{h^2}\psi$$

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2m(E - V_x)}{h^2}\psi = 0$$

This is the Expression for Schrodinger's time-independent equation.

Applications of Schrödinger's equation

Particle in one dimensional potential well of infinite height

Consider particle confined in a one dimensional potential well of infinite height and of length 'L' as shown in the figure 1.5.

- a) The particle move along x - axis between $x=0$ and $x=L$ exhibits elastic collisions

- b) The particle inside the potential well is not influenced by any external force. Therefore potential energy is minimum and taken to be zero.

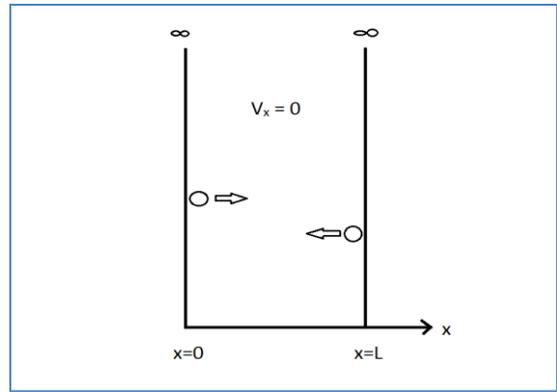
$$V_x = 0 \text{ for } 0 < x < L$$

- c) The potential energy outside the potential well is infinite.

$$V_x = \infty \text{ for } 0 > x > L$$

Consider Schrödinger time independent equation

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2m(E-V_x)}{h^2}\psi = 0 \quad (1)$$



For the particle inside the potential well potential energy

$$\text{i.e. } V_x = 0 \text{ for } 0 < x < L$$

Substituting in (1) we get

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2mE}{h^2}\psi = 0 \quad (2)$$

$$\text{Let } \frac{8\pi^2mE}{h^2} = k^2 \quad (3)$$

Then equation (2) becomes

$$\frac{d^2\psi}{dx^2} + k^2\psi = 0$$

The solution of the above equation is given by

$$\psi(x, t) = A \cos kx + B \sin kx \quad (4)$$

Where A and B are constants depends on boundary conditions

- a) When $x = 0$; $\psi(x, t) = 0$

$$\text{equation (4) becomes } 0 = A \cos 0 + B \sin 0$$

$$A = 0$$

Substituting $A = 0$ in equation (4)

$$\psi(x, t) = B \sin kx \quad (5)$$

- b) When $x = L$; $\psi(x, t) = 0$

$$\text{Equation (5) becomes } 0 = B \sin kL$$

$$\text{On solving } B \neq 0 \text{ or } \sin kL = 0$$

$$kL = n\pi$$

$$\therefore k = \frac{n\pi}{L} \quad (6)$$

Substituting equation (6) in (3), we get

$$\frac{n^2\pi^2}{L^2} = \frac{8\pi^2 m E}{h^2}$$

$$E_n = \frac{n^2 h^2}{8mL^2} \quad (7)$$

The above equation is called the Eigen energy values for a particle in one dimensional potential well.

For $n = 1$
$$E_1 = \frac{h^2}{8mL^2}$$

This is the least possible energy possessed by the particle in a 1-D potential well called Zero-point energy.

For $n = 2$
$$E_2 = 4E_1$$

This is the energy of the particle in first excited state and so on.

Normalization: For the particle inside the potential well, the probability of finding a particle is unity and is given by.

$$\int_0^L |\psi|^2 dx = 1$$

$$\int_0^L B^2 \sin^2 kx dx = 1$$

$$\int_0^L B^2 \left[\frac{(1 - \cos 2kx)}{2} \right] dx = 1$$

$$\int_0^L \frac{B^2}{2} dx - \int_0^L \frac{B^2}{2} \cos 2kx dx = 1$$

On simplification we get

$$B = \sqrt{\frac{2}{L}}$$

Substituting $B = \sqrt{\frac{2}{L}}$ and $k = \frac{n\pi}{L}$ in (5)

$$\psi_n(x, t) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$$

This equation is called as Eigen wave function, where $n = 1, 2, 3, 4 \dots$

Eigen energy values and Eigen function for a particle in infinite potential well:

We have equation for the Energy Eigen values

$$E_n = \frac{n^2 h^2}{8mL^2}$$

and the Eigen wave function is

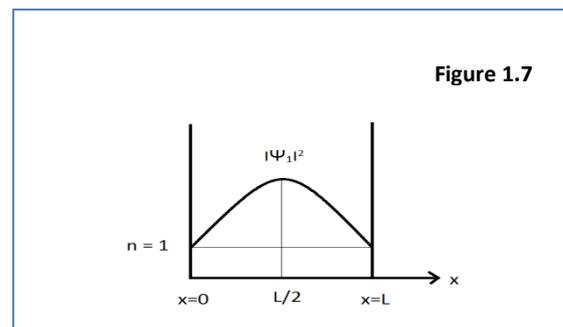
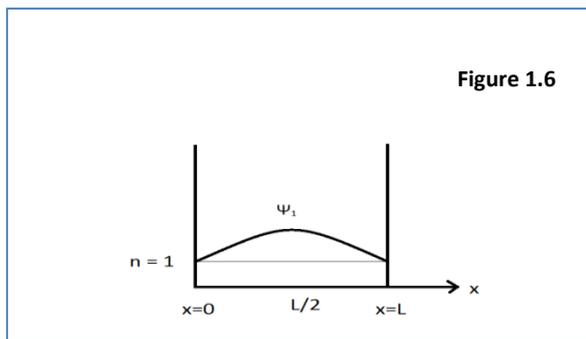
$$\psi_n(x, t) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L} \quad (8)$$

Case i When $n = 1$ for ground state, the wave function and corresponding energy for a particle in ground state is

$$E_1 = \frac{h^2}{8mL^2}$$

$$\psi_1(x, t) = \sqrt{\frac{2}{L}} \sin \frac{\pi x}{L}$$

E_1 is called the ground state energy or Zero-point energy and ψ_1 is the wave function



corresponds to particle in ground state. At the boundary conditions $x = 0$ and $x = L$, the wave function $\psi_1 = 0$. It indicates the probability of finding the particle at both $x = 0$ and $x = L$ is zero i.e., $|\psi_1|^2 = 0$. However, the probability of finding the particle is maximum at the centre i.e., $x = \frac{L}{2}$. The physical representation of the normalized wave function and probability density of wave function of particle at $n = 1$ is shown in below figures 1.6. and figure 1.7.

Case ii When $n = 2$, the particle in the excited state known as first excited state. Therefore, the energy and the wave function corresponding of the particle in the first excited state is given by

$$E_2 = \frac{4h^2}{8mL^2}$$

$$\psi_2(x, t) = \sqrt{\frac{2}{L}} \sin \frac{2\pi x}{L}$$

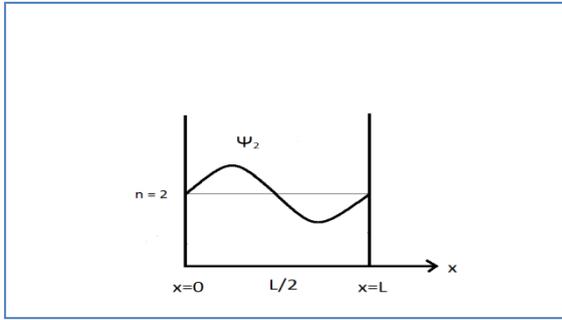


Figure 1.8

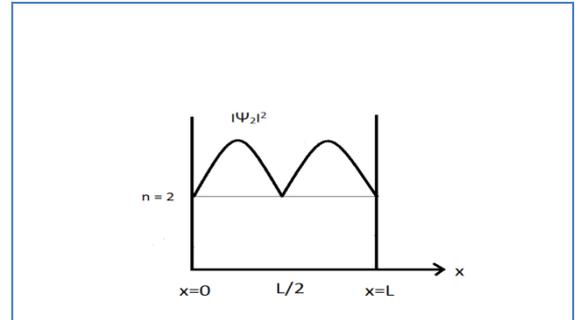


Figure 1.9

The normalized wave function and probability density of wave function of the particle in the first excited state is as shown in the figure 1.8. and 1.9.

The probability of finding the particle in the first excited state at $x = 0$, $x = \frac{L}{2}$ and $x = L$ is zero i.e. $|\psi_2|^2 = 0$. Since $\psi_2 = 0$. However the probability of finding the particle is maximum at $x = \frac{L}{4}$, and $x = \frac{3L}{4}$.

Reference:

1. **Concepts of Modern Physics: Arthur Beiser. McGrawhill company. 2009.**
2. **Modern Physics: Robert Resnic, Robert Eiseberg. John-Wiley.2000.**
3. **Engineering Physics by Rajendran, McGrawhill company. 2011.**

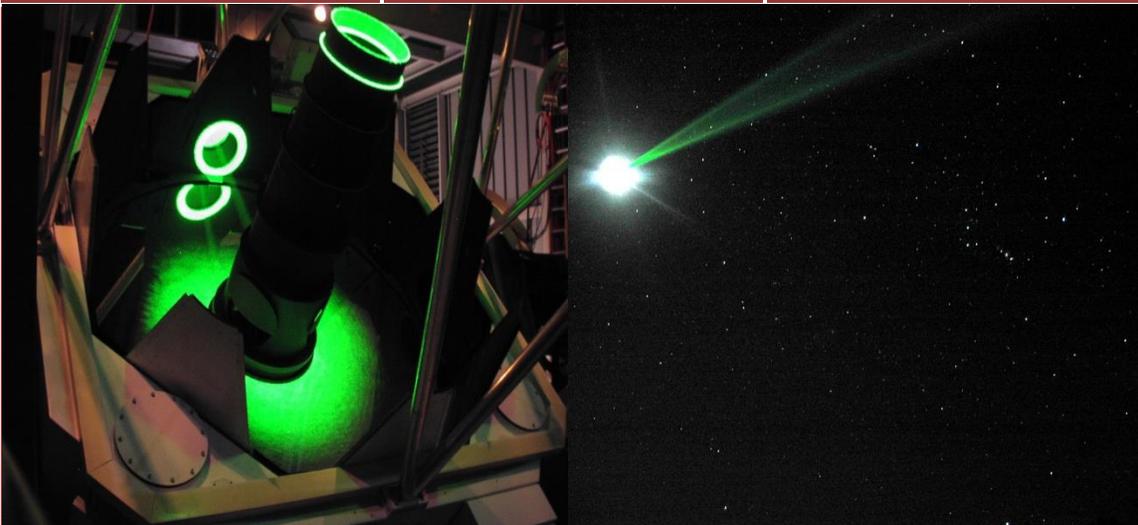
BRINDAVAN COLLEGE OF ENGINEERING

LASERS

Controlled emission of photons...

A R VENUGOPAL

2014



DEPARTMENT OF PHYSICS

LASERS

VTU Syllabus: Principle and production. Einstein's coefficients (expression for energy density). Requisites of a Laser system. Condition for Laser action. Principle, Construction and working of He-Ne and semiconductor Laser. Applications of Laser - Laser welding, cutting and drilling. Measurement of atmospheric pollutants. Holography - Principle of Recording and reconstruction of 3-D images. Selected applications of holography.

Introduction: Lasers are devices that produce intense beams of light which are monochromatic, coherent, and highly collimated. The wavelength (color) of laser light is extremely pure (monochromatic) when compared to other sources of light, and all of the photons (energy) that make up the laser beam have a fixed phase relationship (coherence) with respect to one another. Light from a laser typically has very low divergence. It can travel over great distances or can be focused to a very small spot with a brightness which exceeds that of the sun. Because of these properties, lasers are used in a wide variety of applications in all walks of life. The term "laser" is an acronym for *Light Amplification by Stimulated Emission of Radiation*. To understand the laser, one needs to understand the meaning of these terms. The term "light" is generally accepted to be electromagnetic radiation which interacts with matter results in emission of light, and is discussed briefly with the help of Planck's photon hypothesis and Bohr's atomic model as follows.

Planck's Hypothesis is that, light is made up of particles called "photons" which exhibit both particle-like and wave-like properties. Each photon has an intrinsic energy determined by the equation

$$E = h\nu$$

where ν is the frequency of the light and h is Planck's constant.

According to Bohr's atomic theory, for an atom to absorb light (i.e., for the light energy to cause an electron to move from a lower energy state E_n to a higher energy state E_m), the energy of a single photon must equal, almost exactly, the energy difference between the two states. Too much energy or too little energy and the photon will not be absorbed. Consequently, the wavelength of that photon must be

$$\lambda = \frac{hc}{\Delta E}$$

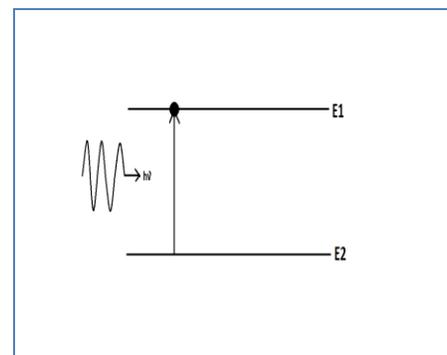
$\Delta E = E_m - E_n$. Likewise, when an electron decays to a lower energy level in a radiative transition, the photon of light given off by the atom must also have energy equal to the energy difference between the two states.

Interaction of radiation with matter

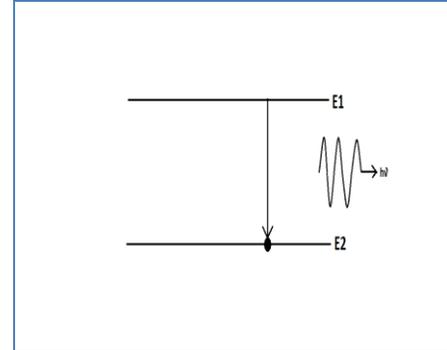
There are three possible ways through which interaction of radiation with matter take place, Absorption of radiation called induced absorption of radiation. Emission can take place in ways by Einstein namely the spontaneous emission and the stimulated emission.

- i) **Induced absorption:** The absorption of incident photon of energy is equal to energy difference two energy states by, a quantum system results in the transition of electron/atom from a lower energy state to a higher energy state.

Consider a two energy state quantum system, with E_1 be the energy of the lower state and E_2 be



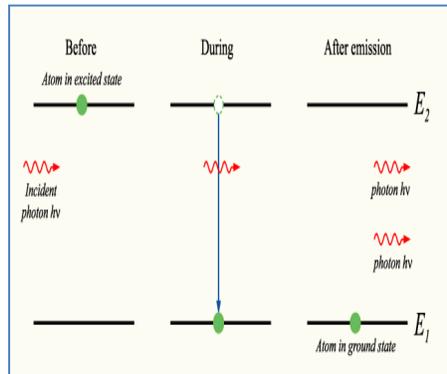
the energy of the higher state as shown in figure 01. Let the photon of energy $\Delta E = E_2 - E_1$ is incident on the system, the electron/atom absorbs the incident photon energy and get excited to higher energy state. Hence, the atom is called the excited atom.



- ii) **Spontaneous Emission:** It is the emission of a photon, when an electron/atom makes transition from higher energy state to lower energy state, without the aid of any external agency.

Consider an electron/atom in the excited state of two energy state quantum system, the excited electron/atom gets de-excited with releasing the energy in the form of photon, with energy. $\Delta E = E_2 - E_1$. Without being aided by any external agency. The emitted photon may not have any phase similarities and can have any direction under identical conditions. Hence they are incoherent.

- iii) **Stimulated emission of radiation:** It is the emission of a photon by a quantum system, when a photon of right energy is incident on the system, due to which the electron/atom makes transition from a higher state to lower energy state. Thus, emitted photon is called the stimulated photon and it will have the same phase, wavelength and direction as that of the passing photon.



Consider an electron in the excited state of two energy level system, Let a photon of energy $\Delta E = E_2 - E_1$ is interacted with the electron/atom makes transition into the lower energy state. The emitted photon will have the same phase, direction and wavelength as that of the incident photon, thus emitted photons are highly coherent.

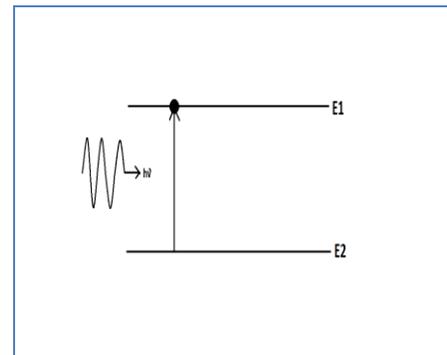
This kind of emission is responsible for laser action.

Expression for Energy density in terms of Einstein's coefficient:

Consider two energy level quantum systems with E_1 be the energy of the lowest energy state and E_2 be the energy corresponds to higher energy state. Let N_1 and N_2 be the number of atoms per unit volume of the system in energy states E_1 and E_2 respectively. Let us consider a stream of photons having the wavelength range λ to $\lambda + d\lambda$ be incident on the system. Then the energy incident per unit volume of the system in the frequency interval of λ and $\lambda + d\lambda$ is given by $E_\nu dv$. Then we can solve for

- i) **Induced Absorption:** In this process, the electron/atom makes transition from lower energy state E_1 to higher energy state E_2 , when it absorbs a photon of energy $\Delta E = E_2 - E_1$ incident on the system. The number of such absorptions per unit time, per unit volume is called rate of induced absorption.

The rate induced absorption depends on the two factors,



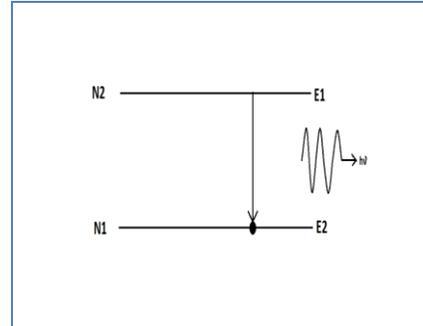
- a) Number of atoms per unit volume in lower energy state i.e., N_1 and
- b) Incident energy density E_ν .

Rate of induced absorption $\propto N_1 E_\nu$

Rate of induced absorption = $B_{12} N_1 E_\nu$

Where, B_{12} is the proportionality constant called Einstein's coefficient of induced absorption.

- ii) **Spontaneous emission:** The spontaneous emission occurs when an electron/atom undergoes transition from higher energy state E_2 to lower energy state E_1 , by emitting a photon of energy $\Delta E = E_2 - E_1$. But this emission is without being aided by any external agency. Hence, it is independent of the incident energy density E_ν and depends only on the number of atoms in the higher energy state N_2 . The number of such emissions per unit time, per unit volume is called the rate of spontaneous emission. Then,

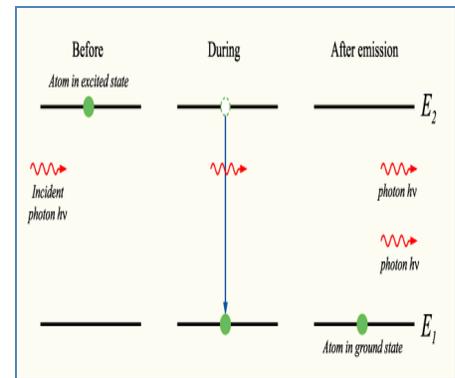


The rate of spontaneous emission $\propto N_2$

The rate of spontaneous emission = $A_{21} N_2$

Where, A_{21} is the proportionality constant called Einstein's coefficient of spontaneous emission.

- iii) **The Stimulated emission:** Stimulated emission requires external photon of right energy $\Delta E = E_2 - E_1$ to stimulate the electron/atom to the downward transition, results in the emission of stimulated photon. Hence, the emitted photons are depends on the incident energy density E_ν , and also depends on the number of atoms in the higher energy state. Then the rate of Stimulated emission $\propto N_2 E_\nu$



Then the rate of Stimulated emission = $B_{21} N_2 E_\nu$

Where, B_{21} is the proportionality constant called Einstein's coefficient of stimulated emission.

Consider the system under thermal equilibrium, under such conditions the number of such photons absorbed per second is equal to the number photos emitted per second both by the spontaneous emission and by stimulated emission processes.

\therefore Under thermal equilibrium,

Rate of Absorption = Rate of spontaneous emission + Rate of stimulated emission

$$B_{12} N_1 E_\lambda = A_{21} N_2 + B_{21} N_2 E_\nu \quad \text{or}$$

$$(B_{12} N_1 - B_{21} N_2) E_\lambda = A_{21} N_2$$

$$E_\lambda = \frac{A_{21} N_2}{(B_{12} N_1 - B_{21} N_2)}$$

Re writing above equation we get,

$$E_\lambda = \frac{A_{21}}{B_{21}} \left(\frac{1}{\frac{B_{12} N_1}{B_{21} N_2} - 1} \right) \text{-----(1)}$$

We have the ratio of population by Boltzmann's law

$$\frac{N_1}{N_2} = e^{\frac{h\nu}{kT}} \text{-----(2)}$$

Substituting Equation (2) in (1), we get,

$$E_\lambda = \frac{A_{21}}{B_{21}} \left(\frac{1}{\frac{B_{12}}{B_{21}} \frac{h\nu}{e^{kT}} - 1} \right) \text{-----(3)}$$

But we have from Planck's radiation law

$$E_\lambda = \frac{8\pi h\nu^3}{c^3} \left(\frac{1}{e^{\frac{h\nu}{kT}} - 1} \right) \text{-----(4)}$$

On comparison of equations (3) and (4) we get,

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3} \quad \text{and} \quad \frac{B_{12}}{B_{21}} = 1 \quad \text{or} \quad B_{12} = B_{21}$$

The Equation for the energy density is

$$E_\lambda = \frac{A_{21}}{B_{21}} \left(\frac{1}{e^{\frac{h\nu}{kT}} - 1} \right)$$

Condition for population Inversion: We know that the average life time of an atom in the unstable state is of the order of 10^{-8} Seconds, during the de-excitation, the spontaneous emission will occur. Since the number of atoms in the lower energy state are greater than that of higher energy state. Hence emitted light is not coherent. The ration of population in the energy states is given by Boltzmann's equation.

$$\frac{N_2}{N_1} = e^{-\left(\frac{E_2 - E_1}{kT}\right)}$$

In general $E_2 > E_1$. Then $N_2 < N_1$

That is, The population of higher energy state is very much less than that of the lower energy state. Hence the probability of spontaneous emission is more than the Stimulated emission. Physically, if $N_2 > N_1$, then, Stimulated emission will take place. The process by which the population of higher energy state is greater than that of lower energy state can be achieved called population inversion.

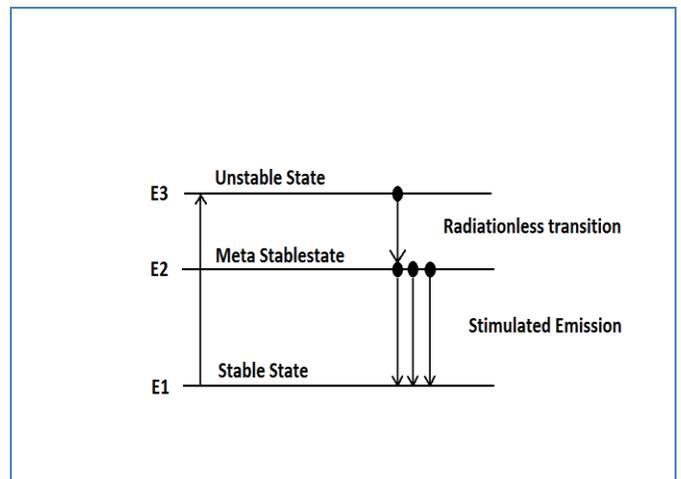
Population Inversion:

A State in which the population of higher energy state is greater than that of lower energy state can be achieved called population inversion.

Consider a three energy level quantum system with E_1, E_2 and E_3 are the energies of the lower, first excited and second excited states respectively as shown in figure.

Let the atoms excited from the stable state E_1 to the unstable state E_3 , Since the life time of the atoms is very small in unstable state. These atoms undergo downward radiation less transition to meta stable state E_2 , and stays over a long time. Hence population of the E_2 state increases steadily, under these conditions a stage will be reached wherein the population of E_2 overtakes that of E_1 , which is known as population inversion.

Once the population of E_2 exceeds E_1 . The stimulated emission is more than that of spontaneous emission and emitted photos will have same phase, wavelength and direction,



grows to a large number and build up the laser light. Hence the condition for laser action is achieved by population inversion.

Basic requirements for the production of lasers

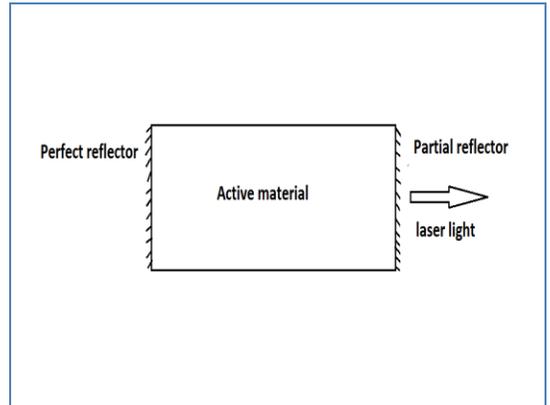
There are three basic requirements for the laser system, and are as follows

- I) **Active material/medium:** The material which is responsible for the laser action, i.e. a material in which amplified signal is attained through population inversion.
- II) **Resonance cavity:** the cavity in which the active material is placed and in which a resonance is brought about by adjusting the two mirrors, such that the distance between them is integral multiples of half wavelength of the amplified signal.

The cavity is in cylindrical in shape having length 'L' with small cross-section, it consists of two mirrors. A fully reflected mirror is placed at one end and a partially reflected is placed at other end. The energy is given to system by the excitation source. The distance between the two mirrors is so adjusted that it is exactly integral multiples of half

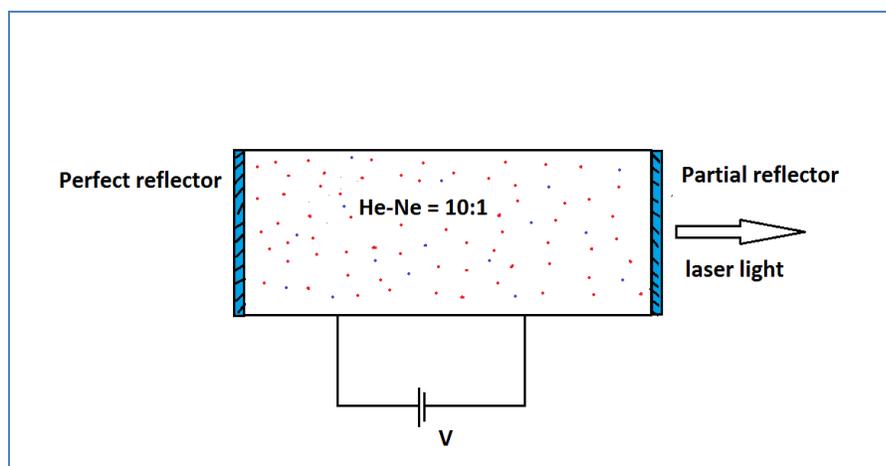
wavelength. This creates the resonance and leads to better stimulated emission. The light is emitted from a narrow of the hole of the partially reflected mirror and becomes unidirectional. The distance between the two mirrors is $= \frac{n\lambda}{2}$. $n=1, 2, 3...$

- III) **Pumping:** The process of exciting the system from lower energy state to higher energy state by means of external source is known as the pumping. If the optical energy is used for excitation then it is called the Optical pumping. When electrical energy is used, it is called electrical discharge.



Helium-Neon Laser:

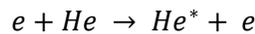
It is a gas laser and basic principle and construction is discussed as follows.



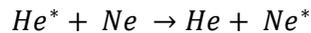
It consists of two main parts.

- i) **Active medium:** The active material used in this laser is a mixture of Helium and Neon gases in the ratio of 10:1. This gas mixture is placed in a gas discharge tube made up of quartz, at the pressure of 1 mm of Hg.
- ii) **Gas discharge tube:** It is made up of fused quartz, it has a length of 80cm and a diameter of 11cm. The end faces of the discharge tube are tilted at the Brewster angle, known as Brewster windows, since the windows are highly transparent for the prepared direction of polarization. A fully reflected mirror is placed at one end of the discharge tube and a partially reflected mirror at the other end.

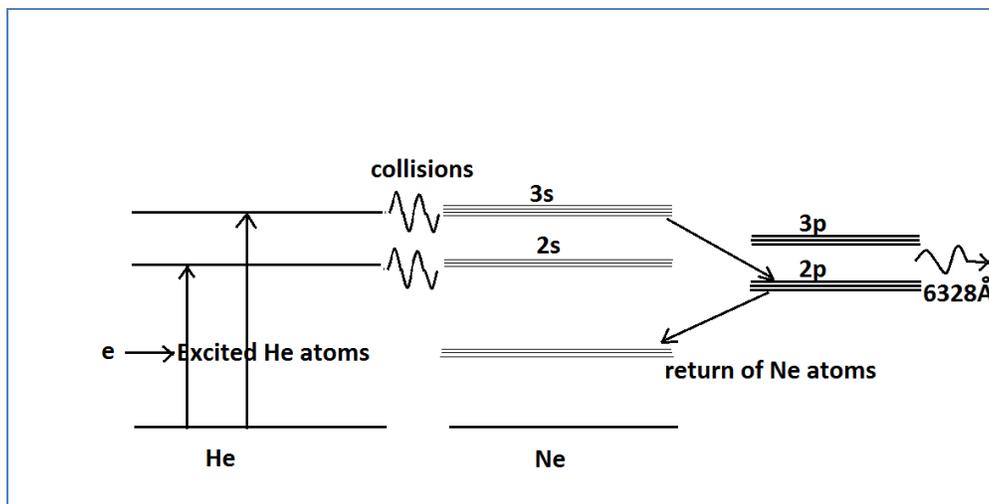
Working Principle: The population inversion for laser action is achieved by inelastic atom-atom collisions due to the electric discharge in the gas mixture. An energetic electron interacts with helium atoms in the ground state. The impact of the electron results in an exchange of energy to the helium atom, as a result, the helium atoms get excited to a higher energy state known as metastable states. This process is known as a collision of the first kind and is represented as follows.



These two energy states are very close in energy to the energy of Neon atom energy states 2s and 3s. Hence, a Helium atom collides with the Neon atoms, making excitation of Neon atoms. This is called a collision of the second kind.



Thus, He atoms help in achieving the population inversion. The excited Ne atoms make a transition from 2s to 2p and 3s to 2p levels for laser action by emitting photons of wavelength 6328Å. This photon travels through the gas mixture parallel to the optic axis of the tube and stimulates surrounding Ne atoms present in the metastable state. This way, a large number of photons will be emitted. These photons travel back and forth between the two parallel mirrors, getting amplified through the stimulated emission every time, finally an intense beam of photons passes through the partially reflected mirror. The emitted photons will have the same phase, wavelength, and directionality.

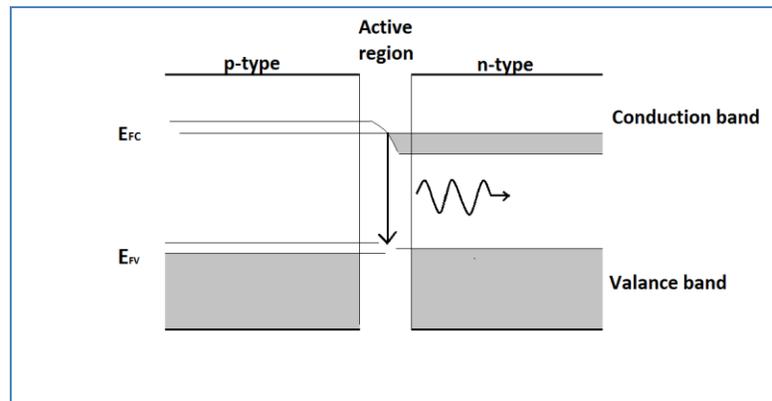


Semiconductor Laser diode:

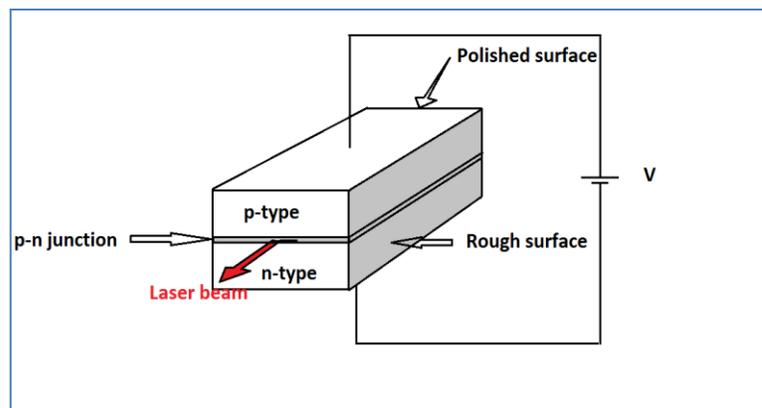
The most compact of all the lasers is the semiconductor laser diode. This laser diode consists of a p-n junction doped in a single crystal of a suitable semiconductor such as Gallium-Arsenide.

Principle: The laser transition can be possible only with the direct band gap semiconductors. Therefore direct band gap semiconducting materials like Ga-As are used for laser action. The p-type material is formed by addition of dopant like tellurium and n-type material is formed by addition of zinc. This heavily doped semiconducting material consists of filled energy states at the bottom of the conduction band and top of the valance band.

When Semiconductor laser diode in the forward biased, holes are injected into p-type and electrons into n-type of the junction. The recombination of electrons and holes within the junction region results in the emission of photons. If the junction current is large enough, population inversion can be obtained between the electron levels and hole levels, that is increasing the population of energy levels near the bottom of the conduction band and top of the valance band. Hence stimulated emission can be achieved.



Working: When the p-n junction is forward biased with the applied voltage equal to the band gap voltage, direct conduction take place. This result in the creation of high current density at the active region, the electron in the conduction band makes the downward transition. These electrons combine with



The holes in valance band and recombination energy are produced as the photon. This photon in turn, may induce another electron in the conduction, there by stimulating another photon. When the applied forward bias voltage is greater than the threshold value, population inversion can be achieved. Hence the stimulated emission leads to the laser action in the semiconductor laser diode.

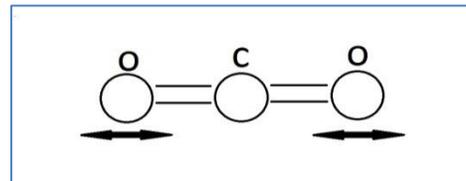
Another requirement for the laser action is the resonance cavity. In the p-n junction device, any of the two end faces are highly polished and another two end faces are rough surfaced to prevent the leakage of photons. The typical Gs-As semiconductor laser diode is as shown in above figure.

Carbon dioxide Laser:

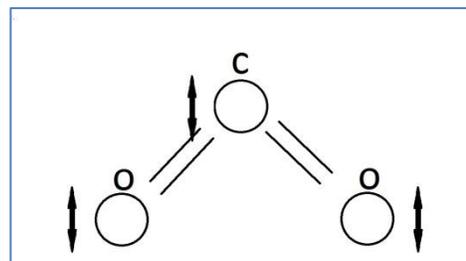
It is the first molecular laser developed by C K N Patel.

Principle: In order to understand the working of this laser, one has to understand the rotational and vibrational spectrum of CO₂ molecules. As the carbon dioxide molecule is linear and Centro symmetric, C atom at center and with linear stretch of O atoms. There are three modes of vibrations and are discussed as follows.

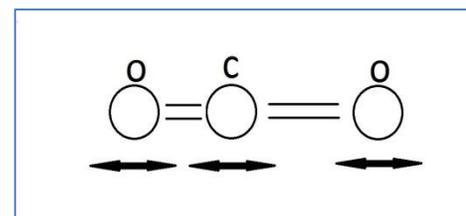
- i. In symmetric mode, the carbon atom is fixed in its position and each oxygen atom stretches symmetrically in linear with the carbon atom known as the symmetric mode of vibrations. Corresponding frequency is known as symmetric stretching frequency
- ii. The oxygen atoms and the carbon atom can vibrate at right angles to the line passing through the Centre of gravity. This is known as bending mode and the corresponding frequency is known as bending frequency.
- iii. The oxygen atoms can vibrate about the central C atom asymmetrically, and at the same time carbon atom also vibrates about its mean position. The corresponding frequency is called the asymmetric frequency.

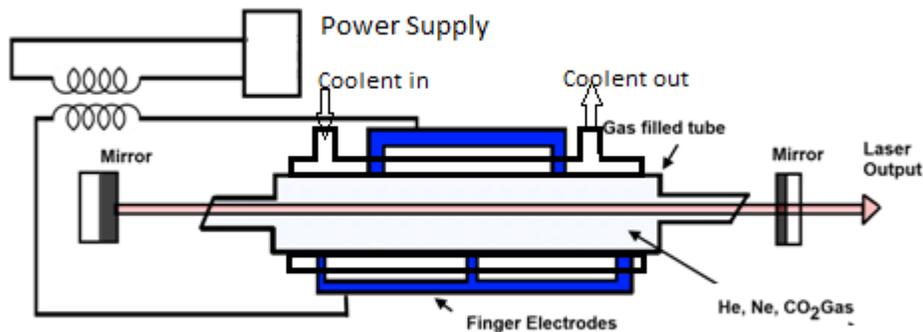


In addition to these three vibrational modes, the molecule can also rotate and therefore rotational energy levels are also possible. Hence , the total energy associated with this molecule is sum of vibrational and rotational energies.

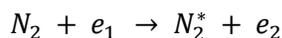


Construction: It consists of discharge tube of length about 5m and of diameter 2.5cm. A mixture of CO₂, N₂ and He in right proportions is enclosed in this tube. The out power of this laser source is increased by increasing the physical dimensions of the discharge tube. The laser is powered by the AC or DC supply. It consist of fully reflected mirror at one and partially reflected mirror at other end. This laser source is water cooled or air cooled to increase the efficiency.

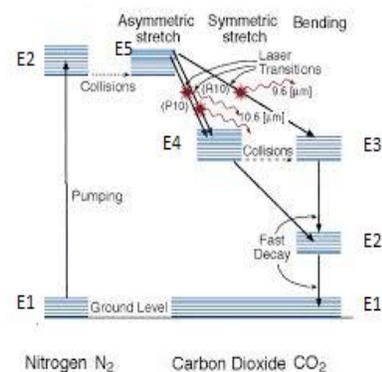
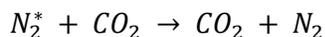




Working: When the suitable voltage is applied across the electrodes, the glow discharge of gas is initiated in the tube. During gas discharge electron will be produced from gas atoms. These free electrons accelerated towards the positive electrode with the collision of nitrogen molecules in their path. The collision of first kind in which energy of the electrons transformed to the nitrogen molecules, makes nitrogen molecules get excited.



The excited N_2^* molecules undergoes collision with CO_2 molecules. Since, the energy values of CO_2 and N_2 are very close, results in second kind of collision, make CO_2 molecules to be excited.



In the metastable state, the population of CO_2 molecules increases rapidly than the other lower energy levels, Thus the population inversion can be achieved, as shown in the below figure.

Once the population inversion is established between the energy states E_5 with respect to E_4 & E_3 energy states. There are two possible transitions take place within the CO_2 molecules energy states.

- i. Transition from E_5 state to E_4 state results in the emission of photons of wavelength $10.6\mu\text{m}$ which is in the infrared region, and
- ii. Transition from E_5 state to E_3 state results in the emission of photons of wavelength $9.6\mu\text{m}$ which is also in infrared region.

Following the above two transitions, the CO_2 molecule in E_3 and E_4 undergoes collisions and de-excited to E_2 state and then to E_1 state by losing its energy. Hence the laser beam will be emitted from partially reflected mirror. The CO_2 laser operates with an efficiency of upto 30%. The power output of few kilowatts can be attained. Hence, generally used in industrial applications.

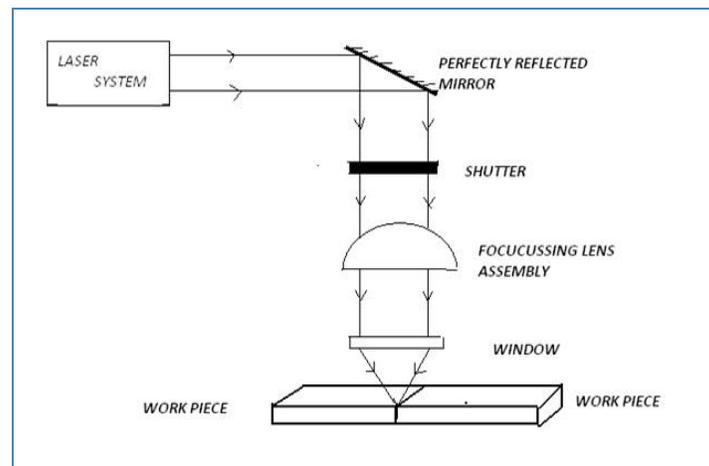
Industrial Applications of Lasers

- i) Laser welding
- ii) Laser Cutting
- iii) Laser drilling

Laser Welding:

Laser welding is a thermal technique, which is used to join the two or more work parts. The joining of metal pieces are established by solidification of metals at common melting point.

Consider two metal plates are to be welded using laser beam, the metal plates are held in contact at their edges as shown in the below figure. The focal spot for the welding on the surface of the metal plate edges are identified. The high intensity laser beam is focused on the spot employing the optical arrangements. At the surface of the metal plate, The highly concentrated laser beam with high intensity is converted into thermal energy and thus heats the spot on the metal plates. Therefore metal plates starts melting and then progress by surface conduction. Thus the metal plates fuse together at the points of contacts. The energy of the beam required to melt the metal plates are kept well below the vaporization temperature of the work piece.



The laser welding process requires few basic components such as a good laser source, a beam guiding, work piece. A Simple experimental setup used for laser welding is as shown in the above figure. The laser light from laser source is made to pass through the optical setup to control the optical energy of the laser beam and to focus the laser beam. At the laser focusing point a shield gas is passed through the shielding gas let. This shielding gas is used to cool the welding spot and to increase the process of welding by convection. Nd-YAG and Carbon dioxide lasers are generally used in the process of laser welding.

Advantages:

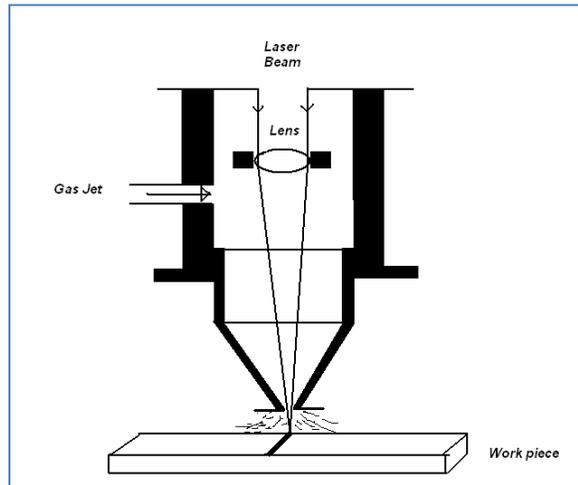
- a) It is a contactless process. Therefore, unwanted materials like oxides can be eliminated.

- b) Only the focused region is heated and so it can be used in micro-electronics, where heat-sensitive components are involved.
- c) There is no mechanical stress on the components involved, thus there is no deformation.
- d) It can be used to weld joints where man cannot physically be present, for example, in nuclear power plants.

Laser Cutting:

The laser cutting is thermo-mechanical process. During the thermal mechanical process, the material is removed either by phase changes like melting and vaporization or by hydrothermal motions.

The below figure shows the typical laser cutting process system. It consists of nozzle with laser beam focusing system and gas inlet to assist the laser cutting. The high intensity laser beam is focused through the lens via nozzle. The high intensity optical beam is focused at the cutting spot, which converts into thermal energy with temperature above the vaporization temperature. Hence, metal starts melts, this molten material will be ejected by providing the necessary mechanical force from inlet gas jet through the nozzle. The passed gas also used to cool the cutting spot by forced convection. On the other hand, one can use the inert gas in order to reduce the oxidation during the laser cutting process. Hence, laser process can be performed with good quality and precisely.



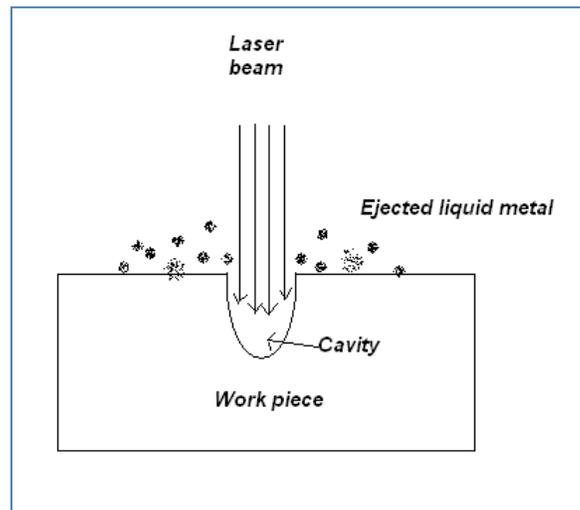
Advantages:

- a) There is no thermal damage and chemical change in the material
- b) There is no wear and tear and no mechanical stress induced
- c) There is no need for a coolant while cutting the material
- d) The cutting is clean, fast, accurate, and of a high quality
- e) Saves manpower and time

Laser Drilling:

The high intensity laser beam is used to obtain the small holes in materials at high speed with good quality. The high intensity laser beam is used to drill the material, a large number of physical processes, ablation mechanism, etc. take place depending on used material and irradiation conditions.

The setup of the laser drilling is as shown the below figure. The high intensity pulsed laser beam is made to incident at the drilling spot on surface of the metal. The duration of the pulses will be of 0.1 ms to 1 ms. The intense heat generated over a short duration by the pulses evaporates the material locally. Hence the hole is left. Nd-YAG Laser is used in case metals but Carbon Dioxide Laser is used in case of both Metals and Nonmetals.



Advantages:

- a) No wear and tear of tools
- b) Drilling can be done at any oblique angle
- c) Fine holes of diameter 0.2 to 0.5 mm can also be drilled adjacent to each other
- d) Even hard or brittle materials can be drilled as there are no mechanical stresses induced in the material

Application of Laser in Measurement of Pollutants in the Atmosphere

Lasers are used for measuring pollutants like carbon monoxide, sulphur dioxide, nitrous oxide, Freon, and a number of other particles which come under the categories of dust, smoke, fly ash and so on. The process of measurement of pollutants by lasers is called **Light Detection and Ranging** or **LIDAR**.

LIDAR system consists of pulsed laser source in the transmitting part and a transceiver optical system, a signal processing unit and a reflector at the receiving part.

In this method, a laser beam from the source is projected through the atmosphere. The laser beam undergoes scattering by the pollutants. At the same time there is absorption of certain amount of laser light by pollutants.

There are two techniques used:

1) Absorption technique:

In this technique, the laser beam is passed through the atmosphere and the absorption spectrum is recorded. Since different elements of a pollutant absorb laser energy at different wavelengths, the study of absorption spectrum indicates the type of pollutant and its amount in the atmosphere.

2) Raman back scattering technique:

In this technique, the laser beam is passed through the atmosphere and the spectrum of transmitted beam is recorded. In this type of diffraction it is Raman scattering that takes place and the spectrum consists of several lines of different intensity. By measuring the change in wavelength of the incident laser beam and the scattered beam, Raman Shift can be calculated. The composition of the pollutants can be assessed by studying the Raman shift.

HOLOGRAPHY

The technique of recording the three dimensional image of an object using the principle of interference is known as holography. The word holography is derived from the Greek words “holos”, meaning whole or complete, and “graphos”, meaning writing.

The recorded three-dimensional image is called a hologram. Holography was invented by **Dennis Gabor** in 1947, when he was trying to improve the resolution of microscopes. Holography requires highly coherent laser light.

In conventional photography, the photographic film records only the intensity of light, and not the phase of the wave. Thus it loses the three dimensional nature of the object. But in holography we record both intensity and phase of the wave by making the wave front from the object to interfere with another wave front from a reference source and recording the interference fringes on a conventional photo film. This result in a 3-dimensional picture of the object called a hologram.

Holography involves two steps:

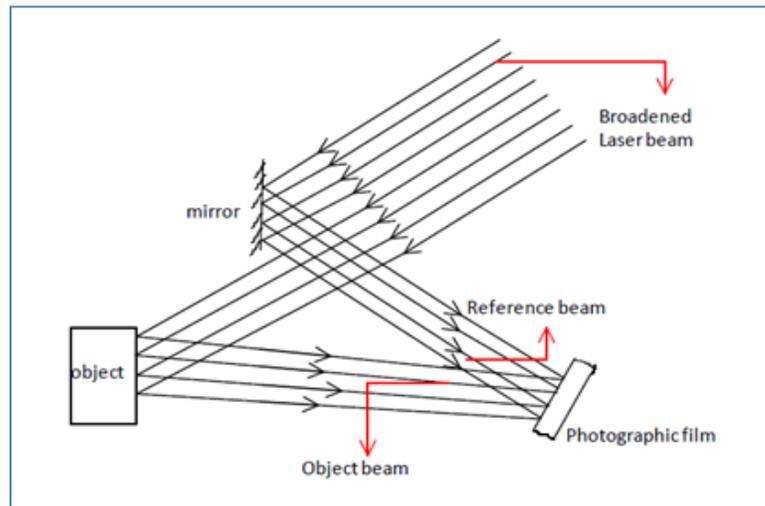
1) Recording of the image of an object

The image of an object can be recorded on a photographic plate using the laser light using two techniques:

A) Wave front division technique

In this method, a broadened laser beam from a laser source is made to fall on the object and a mirror, placed one beside the other as shown in the figure. The reflected rays from the mirror and the object are received on a photographic plate at a suitable distance.

The laser waves reflected from the mirror have plane wave fronts, whereas the waves reflected from the object have spherical wave spherical wave fronts, as each point on the body acts as a point source.



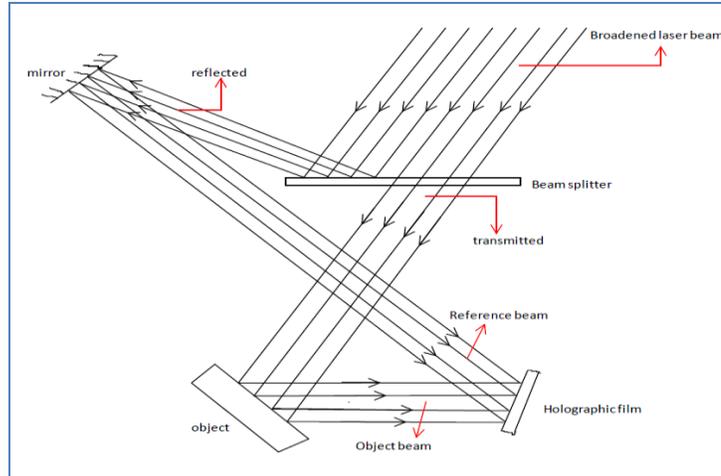
The beam of light reflected from the mirror is called the reference beam and the beam reflected from the object is called the object beam.

The two beams superpose on the photographic plate. The interference pattern due to the interference of the plane wave front and the spherical wave front is recorded on the photographic plate. The developed photographic plate is called Hologram.

B) Amplitude division technique

In this technique, a broadened laser beam is incident on a beam splitter. The beam splitter reflects a part of the incident beam on to a mirror while transmitting the rest to incident on to the object kept at a certain distance from the mirror.

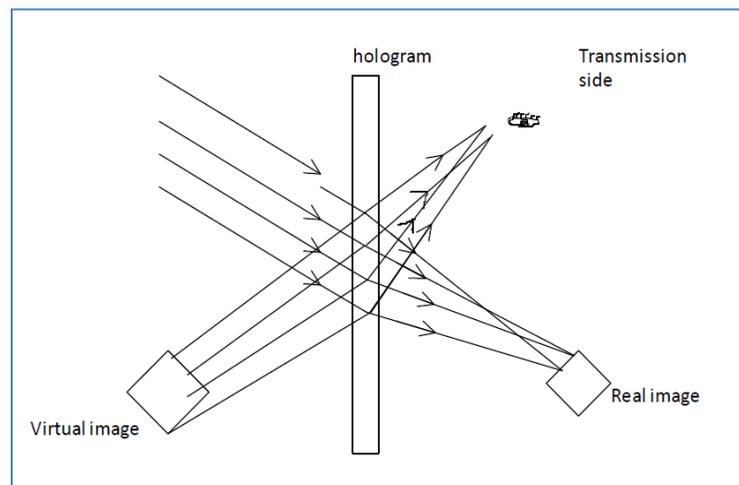
The beam reflected from the mirror is called the reference beam, and the beam reflected from the object, the object beam. These two beams interfere on the photographic plate and an interference pattern is recorded. The recorded photographic film, when developed gives the hologram. This method is considered division of amplitude because the transmitted and reflected light beams from the mirrors have different intensities and hence different amplitudes.



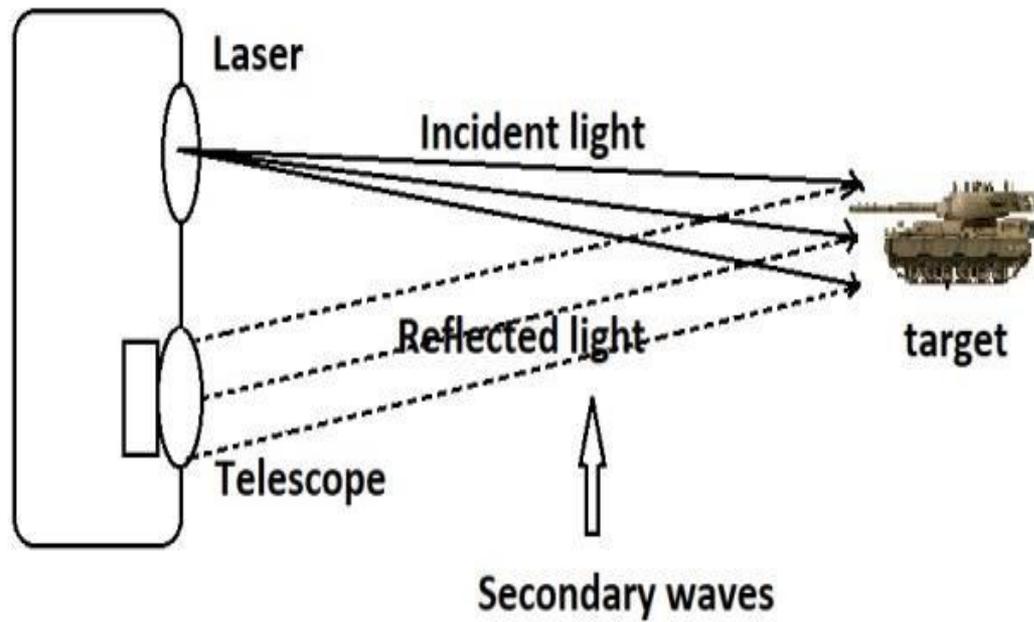
2) Reconstruction of Image

Reconstruction of the holographic image is the process of extraction of 3-dimensional information about the object.

For the reconstruction of the image, the hologram is illuminated by the laser light in the direction in which the reference beam was incident on it during recording. The incident laser beam undergoes diffraction in the hologram and the diffracted waves undergo interference to generate the real image of the object on the transmission side of the hologram, and a virtual image behind the hologram at the original site of the object. When an observer sees the hologram from the transmission side, he receives the rays diverging from the virtual image and it appears as though the original object is lying on the other side.



Laser in defense (laser range finder)



Laser rangefinder in defense:

- ❖ A high powered laser beam from a solid state laser source such as Nd – YAG laser is directed towards enemy target from a transmitter.
- ❖ The pulses are narrow with high peak power, incident on the target and produces reflection,
- ❖ this reflected rays are called secondary rays, and are received by receiver consists of a narrow band optical filter tuned to the frequency of laser light, that all background noise could filtered.
- ❖ The signal will be amplified by Photo-multiplier. The high speed clock in rangefinder measures the exact time 't' of secondary waves reflecting from the target and converts it into distance using a simple equation

$$d = c * t$$

MEDICAL APPLICATIONS OF LASERS (LASIK (laser-assisted in situ keratomileusis))

- ❖ Laser eye surgery is a medical procedure that involves the use of laser to reshape the surface of the eye.
- ❖ This is done to improve or correct myopia (short-sightedness), hypermetropia (long-sightedness) and astigmatism (uneven curvature of the eye's surface).
- ❖ It can also be helpful in overcoming presbyopia (difficulty seeing up close as we get older).
- ❖ During laser eye surgery, a computer-controlled excimer laser is used to remove microscopic amounts of tissue from the cornea.
- ❖ The aim is to restore normal eyesight, without the need for glasses or contact lenses.
- ❖ In one of the operations using the excimer laser, the thin outer layer of the cornea (called the corneal epithelium) is removed and the underlying layers are reshaped. This procedure is known as photorefractive keratectomy (PRK).

Skin treatment

- ❖ Today, there are more medical and aesthetic dermatological procedures utilizing laser technology than ever before.
- ❖ Applications range from mildly invasive procedures such as laser-assisted lipolysis to completely non-invasive ones such as photo biomodulation.
- ❖ This wide spectrum includes skin resurfacing, non-invasive body contouring, hair and tattoo removal, treatment of vascular and pigmented lesions, to name a few.
- ❖ Success in each of these applications is largely dependent on correct choice of laser type, wavelength, and pulse width

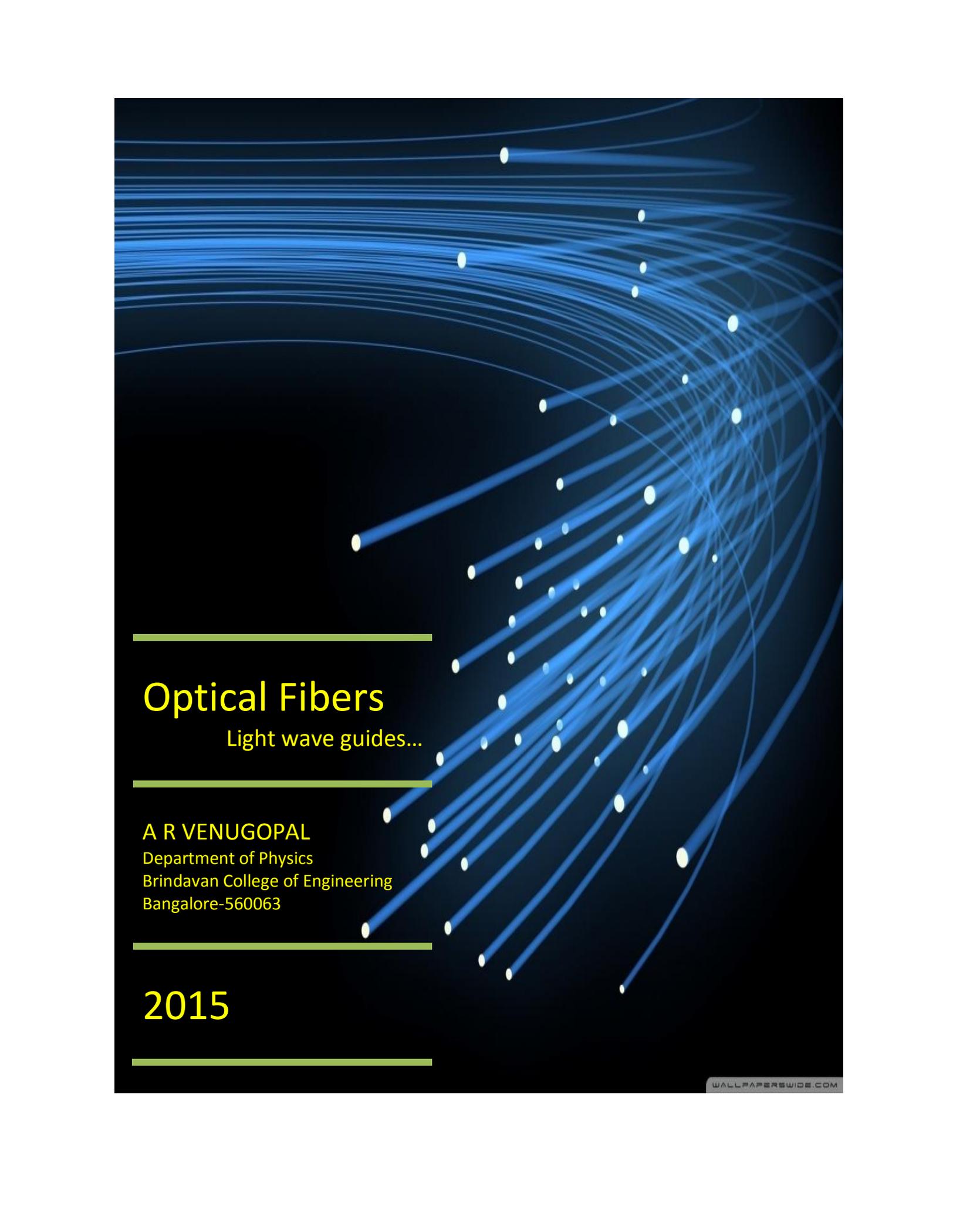
Application of Lasers...

- Laser beam is used to measure distances of sun, moon, stars and satellites very accurately.
- It can be used for measuring velocity of light, to study spectrum of matters, to study Raman effect.
- It can be is used for increasing efficiency of computer.
- It is used for welding.
- It is used in biomedical science.

□ Application of Lasers...

- It is used for communication, T. V. transmission, to search the objects under sea.
- It can be used to predict earthquake.
- Laser tools are used in surgery.
- It is used for detection and treatment of cancer.

- **It is used to align straight line for construction of dam, tunnels etc.**
- **It is used in holography.**
- **It is used in fiber optic communication.**
- **It is also used in military, like LIDAR.**
- **It is used to accelerate some chemical reactions.**
- **It is used in 3D photography.**



Optical Fibers

Light wave guides...

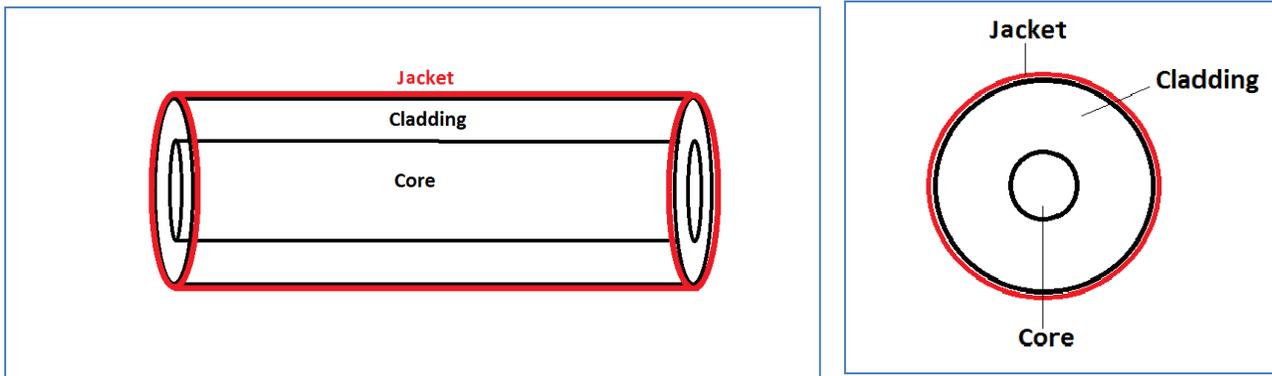
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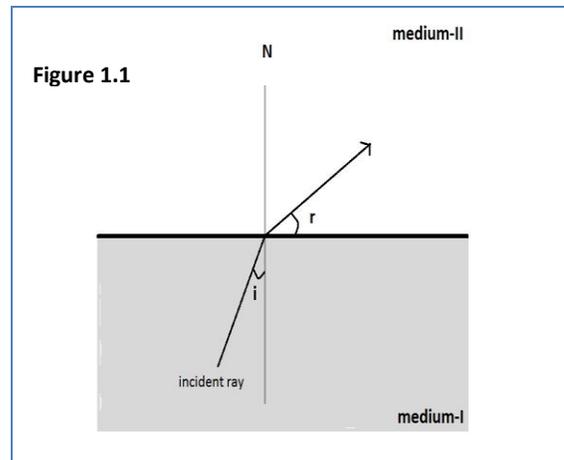
2015

VTU Syllabus: Optical Fibers: Propagation mechanism in optical fibers. Angle of acceptance. Numerical aperture. Types of optical fibers and modes of propagation. Attenuation. Applications - block diagram discussion of point to point communication.

Optical fibers: Optical fibers are cylindrical light wave guides, which guides the light waves to propagate through it without much energy loss, by the principle of total internal reflection. Optical fibers are widely used in fiber-optic communications, which permits transmission over longer distances and at higher bandwidths (data rates) than other forms of communications. It consists of a core and cladding, made up of transparent dielectric materials like glass and plastic. In optical fibers core is cylindrical in shape and is very thin where the light travels and it has higher refractive index. Cladding is an outer optical material surrounded by core; it has lower refractive index than core. Hence the light reflects back into the core by total internal reflection. Jacket or the sheath is the outermost section which protects the fiber from damage and moisture.



Propagation of light in optical fiber: Consider a ray of light propagating from a medium of higher refractive index n_1 into medium of lower refractive index n_2 at the interface by making an angle ' i ' to the normal. The refracted ray bends away from the normal by making an angle ' r ' as shown in the figure 1.1. If the angle ' i ' of incident ray increases and if it becomes equal to critical angle θ_c then the refracted ray propagates through boundary between two media by making an angle 90° as shown in the figure 1.2. When the angle ' i ' of incident ray exceeds the critical angle θ_c , The incident ray gets reflected back into the same medium. Hence there is no loss of intensity of refracted ray as shown in figure 1.3. *The reflection of light into the same medium for the incident angle greater than critical angle is known as total internal reflection.* Therefore total internal reflection is the basic principle involved in the transmission of signals in the optical fibers.



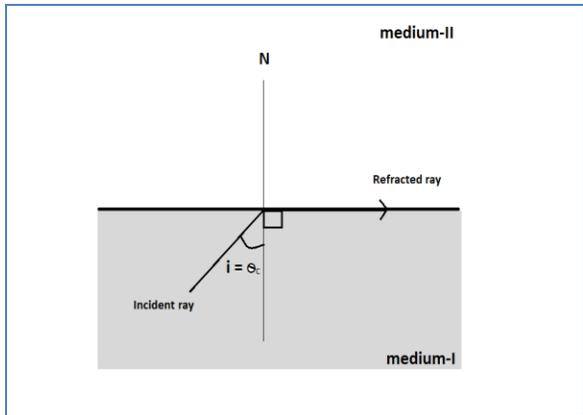


Figure 1.2

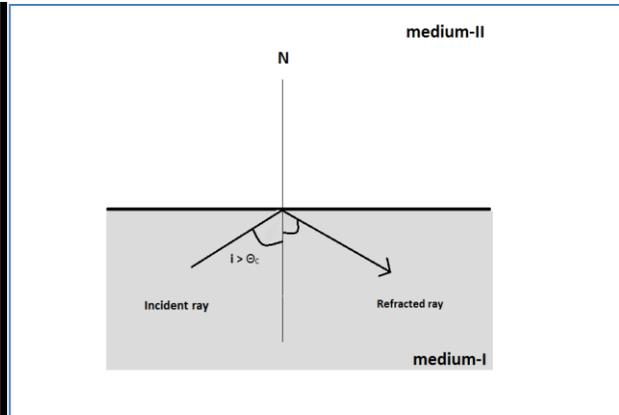


Figure 1.3

Applying the Snell's law for the incident ray shown in the figure 1.2

$$n_1 \sin i = n_2 \sin r$$

For $i = \theta_c$ and $r = 90^\circ$. The above equation simplified to

$$n_1 \sin \theta_c = n_2 \sin 90^\circ$$

$$n_1 \sin \theta_c = n_2$$

$$\theta_c = \sin^{-1} \left[\frac{n_2}{n_1} \right]$$

The above equation determines the critical angle for an optical fiber.

Acceptance angle and Numerical aperture for an optical fiber:

In order to transmit the light wave through the optical fiber, it is necessary to launch the light at angles that fall within certain range. The maximum limit of this angle is decided by the acceptance angle.

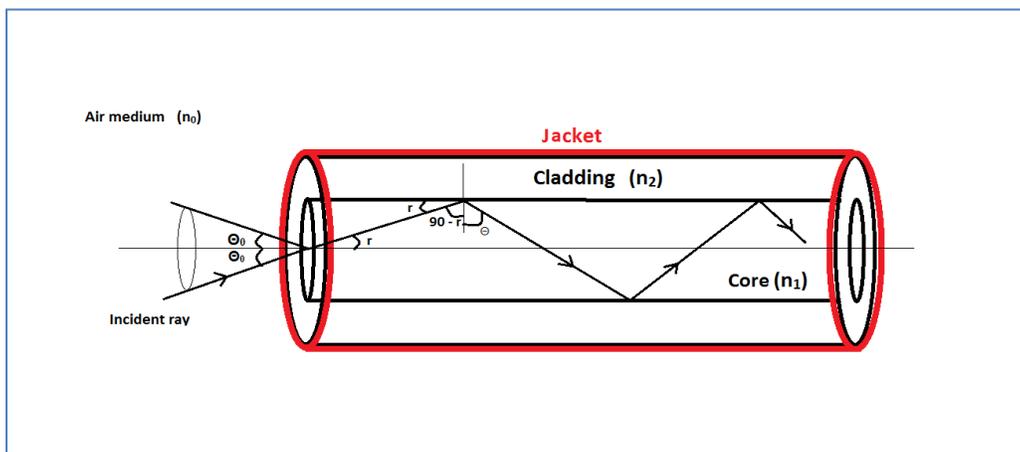


Figure 1.4

Let us consider an optical fiber surrounded by air medium of refractive index n_0 , into which light is transmitting at an angle θ_0 at core-air medium interface, it undergoes refraction through core of refractive index n_1 at an angle 'r' with fiber axis. This refracted ray is incident at core-cladding interface at greater than the critical angle of $90 - \theta$. Therefore the ray undergoes total internal reflection by making an angle θ as shown in the figure 1.4. The refractive index of cladding is considered to be n_2 ($n_2 > n_1$). Therefore the light gets totally internally reflected for the transmission along the length of the optical fiber.

Applying the Snell's law to the launching face of the fiber, we get

$$\begin{aligned} n_0 \sin \theta_0 &= n_1 \sin r \\ n_0 \sin \theta_0 &= n_1 \sqrt{1 - \cos^2 r} \quad (1) \end{aligned}$$

But for air medium $n_0 = 1$, we get

$$\sin \theta_0 = n_1 \sin r \quad (2)$$

Now, Applying the Snell's law at core-cladding interface

$$n_1 \sin(90 - r) = n_2 \sin 90$$

For the angle of incidence is equal to critical angle the refracted ray passes through core-cladding interface by making angle 90° with normal.

$$\begin{aligned} n_1 \cos r &= n_2 \\ \text{Or} \quad \cos r &= \left[\frac{n_2}{n_1} \right] \quad (3) \end{aligned}$$

Substituting equation (3) in (1) we get

$$\begin{aligned} n_0 \sin \theta_0 &= n_1 \sqrt{1 - \left[\frac{n_2^2}{n_1^2} \right]} \\ \sin \theta_0 &= \sqrt{\left[\frac{n_1^2 - n_2^2}{n_0^2} \right]} \\ \theta_0 &= \sin^{-1} \sqrt{\left[\frac{n_1^2 - n_2^2}{n_0^2} \right]} \end{aligned}$$

This is an expression for acceptance angle or half cone acceptance angle for an optical fiber.

The light incident within the half cone acceptance angle, the refracted rays undergo total internal reflection in the core part of an optical fiber. The light ray can propagate through the optical fiber only when the angle of incidence is less than the acceptance angle.

$$\theta_i < \theta_0 \quad \text{or} \quad \sin \theta_i < \sin \theta_0 \quad \text{or} \quad \sin \theta_i < NA$$

Numerical aperture (NA) is the most important parameter of an optical fiber. It is the measure of how much light can be collected by the fiber for the transmission. It is the sine of the half cone acceptance angle, and is given by.

$$\sin \theta_0 = \sqrt{\left[\frac{n_1^2 - n_2^2}{n_0^2} \right]}$$

$$NA = \sqrt{\left[\frac{n_1^2 - n_2^2}{n_0^2} \right]}$$

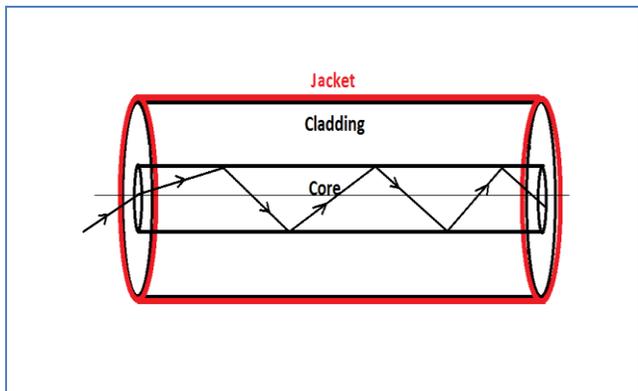
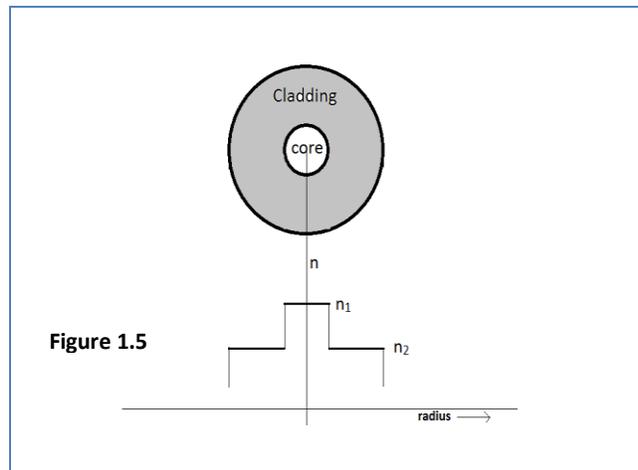
This relation shows the light gathering ability of an optical fiber increases with its numerical aperture.

Types of optical fibers

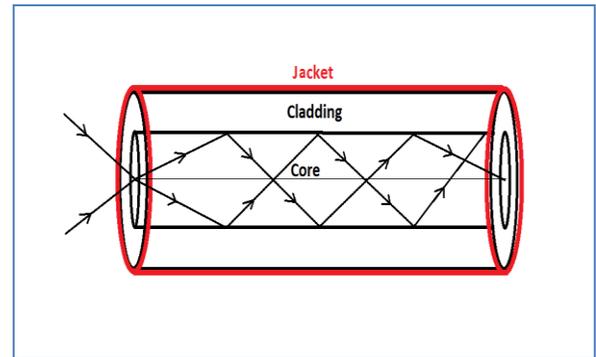
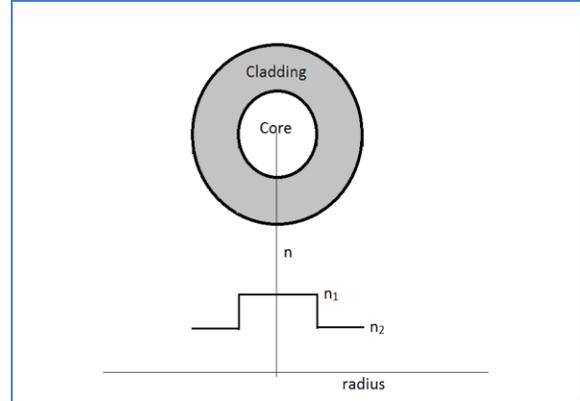
Based on the number of modes transmitted into fibers, the optical fibers are classified into two types namely,

- i) Single mode and
- ii) Multi mode

Single mode optical fiber: Single mode optical fibers are fibers which transmit signals in single mode during transmission. The diameter of core is of the order of 5 - 10 μm and that of cladding is less than 100 μm . Single mode fibers have a lower signal loss and higher transmission rate or higher band width than multimode fibers. Also these fibers have lower dispersion. In general, single mode optical fibers are considered to be low loss fibers. So these fibers are most useful for large bandwidth applications. Since these fibers are more resistant to attenuation, they can also be used in significantly longer cable runs. The refractive index profile of single mode is shown in figure 1.5.

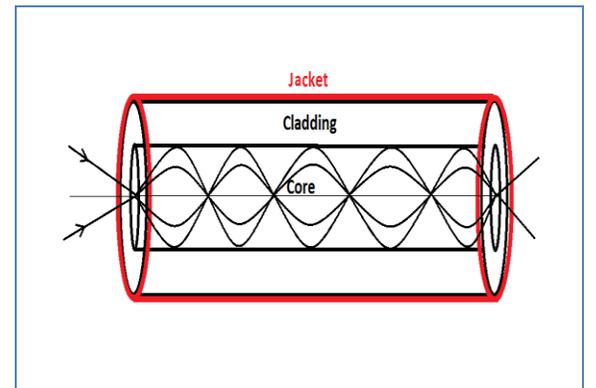
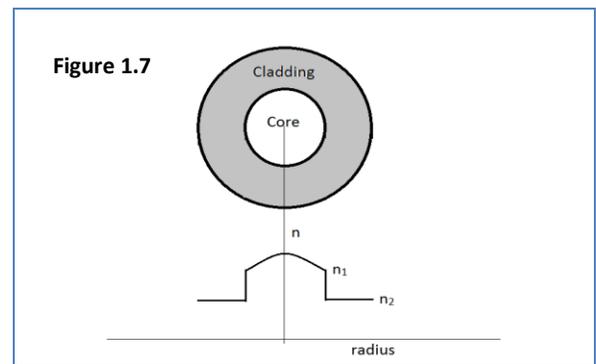


Multimode Step Index optical fiber: It allows more than one to propagate. The diameter of the core is of the order of $50 - 100\mu\text{m}$ and diameter of cladding is less than $200\mu\text{m}$. Multimode fibers used in short lengths, such as Local Area networks (LANs) and Storage Area Networks (SANs). Because the multimode optical fiber has higher NA and larger core size, fiber connections and launching of light is very easy. Due to several modes of transmission the dispersion increases, i.e. the modes arrive at the fiber end at slightly different times so spreading of pulses take place. The refractive index profile of multimode fiber is shown in the figure 1.6.



Multimode Graded Index optical fiber:

The geometry of the graded index optical fiber are same as that of multimode step index fibers as shown in figure 1.7. The refractive index of core decreases with the increasing the radial distance from the fiber axis and become same at core cladding interface and refractive index of cladding is uniform. The change in the refractive index of core with increases in radius makes the light signals to travel at the outer regions of the core than the central region. Thus the dispersion of the light signals can be minimized with this type of fiber design. Under these conditions, the transmitted signals follow the sinusoidal paths along the fiber. The advantage of graded index fiber in comparison with the multimode step index fibers is considerable decrease in the modal dispersion.



Fractional index change: Is defined as the ratio of the refractive index difference between the core and cladding to the refractive index of the core.

$$\Delta = \frac{n_1 - n_2}{n_1}$$

Or

$$\Delta n_1 = n_1 - n_2$$

We have, numerical aperture

$$NA = \sqrt{\left[\frac{(n_1 - n_2)(n_1 + n_2)}{1} \right]}$$

$$NA = \sqrt{\Delta n_1 (n_1 + n_2)}$$

But $n_1 \approx n_2$

$$NA = \sqrt{\Delta 2n_1^2}$$

V – Number: The number of modes supported by an optical fiber is described by a parameter called the V – number. For an optical fiber of radius 'r' carries a signal of wavelength 'λ'. Then the number of modes supported by the optical fiber is given by

$$V - Number = \frac{2\pi r}{\lambda} \sqrt{n_1^2 - n_2^2}$$

$$V - Number = \frac{2\pi r}{\lambda} (NA)$$

Then the total number of modes supported by an optical fiber is given by $N = \frac{V^2}{2}$

Attenuation: The loss of intensity of the transmitted signals through the optical fibers is called attenuation. This alters the transmission property of the optical fiber. When the optical signal is transmitted into core, the loss of intensity is mainly due to the following.

- i) Absorption
- ii) Scattering
- iii) Radiation losses.

Absorption: Optical fibers are made up of glass polymers and hence the absorption is resulted due to

1. Defects in the composition
2. Impurities in the material (extrinsic absorption)
3. Characteristic absorption of the material of the fiber (intrinsic absorption)

Apart from the above factors the amount of attenuation happens because of OH ions present and the existence of transition metals. The presence of OH ions cannot be avoided because the hydrolysis process involved in the preparation of optical fibers. Thus, in the transmission of signal

through the optical fibers, care has to be taken not to use the wavelength of light where absorption occurs due to OH ions and the characteristic peaks of the material of the fiber appear.

Scattering loss:

However best the material may be, it is impossible to have its density uniform through out. The variation in the density leads to the variation in the refractive index, of course to a very little extent; even this little variation causes refraction resulting into attenuation. The complexity increases because of the variation in the refractive index with wavelength also. Thus, the scattering induced attenuation, which is a Rayleigh's type of scattering of light, is very complex in nature.

Structural in-homogeneities and defects created in the fiber fabrication can also cause scattering. This scattering being dependent on value of wavelength and it decreases with increase in wavelength. Thus, using higher wavelengths one can reduce this type of losses.

Radiation losses:

1. Bending losses

Because of the bending of the fiber arising out of its installation from one point to another which are known as macroscopic bending, that is bending because of the curvatures in the fiber also leads to attenuation. Another type of bending arising out of uneven forces on the interface leading to change in the refractive indices of core and clad resulting in to microscopic losses. These can be reduced by having high radius of curvatures at the time of cabling and by using a high ordered clad which has a good resistance for small variation in external forces.

2. Core and cladding losses:

Since core and cladding have to have different refractive indices and their multiplicity being high in step index multimode and graded index fibers, where more than one material is used, the attenuation coefficients are going to be different. This in itself creates scattering and hence attenuation when ray of light tries to travel from one medium to another.

3. Dispersion loss:

The dispersion in the light is obvious because of the refractive indices of the different wavelengths being different in the transmission. This arises out of the characteristic of the material. Because of this the angle of incidence for different wavelengths will be different when a ray tries to travel towards the clad resulting in to uneven total internal reflection and sometimes transmission of light from core to clad. This type of attenuation thus arises because of the wavelength itself.

The different attenuation discussed above is because of the some important parameters of the material of fiber and its construction; however the attenuation also happens because of group delay pulse broadening etc which are of not much importance for ordinary applications. However, they play a very important role in case of signal processing. A major loss of signal occurs because of the coupling between two optical fibers, although it cannot be considered as an attenuation problem for an individual fiber but it becomes a major problem of attenuation when whole of the system is considered.

Expression for attenuation coefficient:

Consider an optical fiber of length 'l' carries an optical signal. Let I_i be the intensity of the input signal at transmission end of an optical fiber and I_o be the intensity of the signal at the receiving end after travelling through a distance of 'L'. The intensity of the transmitted signal decreases due to attenuation. According to Lambert's law the decrease in the intensity of the transmitted signal with the increase in the distance is proportional to Intensity of the signal.

$$\text{i.e.} \quad -\frac{dI}{dl} \propto I$$

The negative sign indicates the decrease in the intensity of the transmitted signal

$$I = -\alpha \frac{dI}{dL}$$

Where α is the attenuation coefficient

Re-writing the above equation,

$$dl = -\alpha \frac{dI}{I}$$

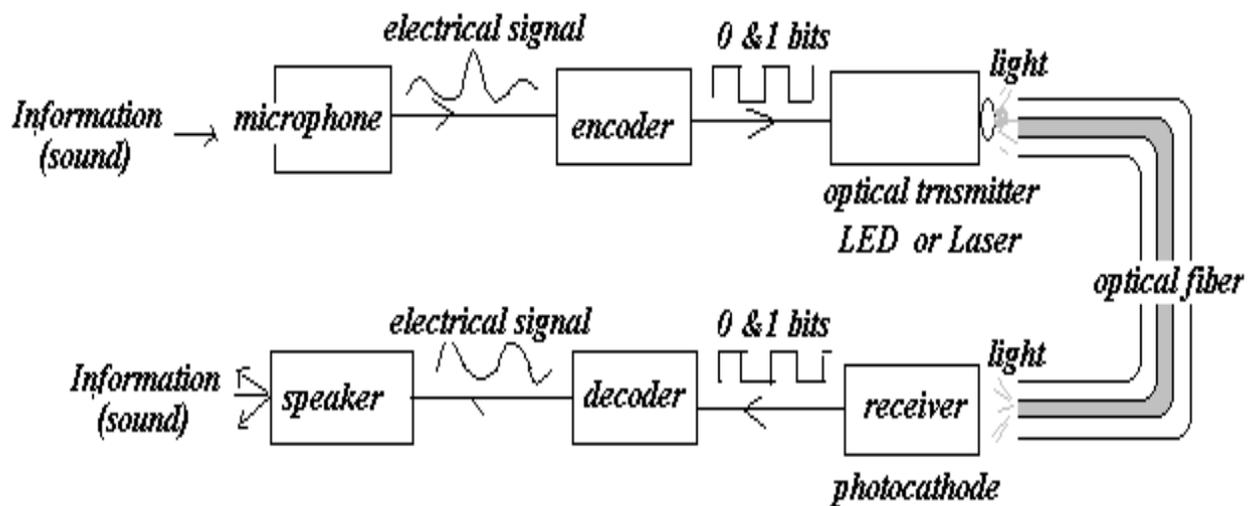
Integrating the above equation with limits $l = 0$ to $l = L$ and the intensity I_i to I_o we get

$$\int_{l=0}^{l=L} dl = -\alpha \int_{I_i}^{I_o} \frac{dI}{I}$$
$$\alpha = \frac{10}{L} \log_{10} \frac{I_i}{I_o} \quad \text{dB/km}$$

This is the expression for the attenuation coefficient in an optical fiber.

Application of optical fibers for communication

Point to point communication: The optical fibers are widely used in various fields like communications, Medical, Domestic etc. One of its applications in point to point communication is discussed in detail here



In optical fiber communication the signal can be transmitted through the fiber only in the form of optical signal. For which first the audio signal is converted into electrical analog signals with the help of transmitter in the telephone and these analog signal are fed in to a coder where they get converted in to binary form. This binary data is made to enter in to an optical transmitter where these get converted in to optical signals by modulating the light signals. These optical signals are fed in to the optical fiber within the angle of acceptance. At the output end the optical signals are fed in to a photo detector which converts them in to Binary form and then in to a decoder where the signal converted in to analog form and finally gets converted in to voice at the receiver end. As the signal propagates through the optical fiber there may be loss of signal due to various factors like as discussed in attenuation and also the signal may be distorted due to the spreading of pulses with time which is mainly because of the variation in the velocity of the different spectral components through the optical fiber. With distance the loss of the signal becomes more. Hence before the entire gets lost it is necessary to have repeaters which amplifies the signal but there is no such device which can directly amplify the optical signal and therefore at each repeater the optical signal needs to be converted in to electrical signal then amplify and again convert it back in to optical signal and then fed in to the fiber. This process has to be followed at every repeater, which restricts the speed of the signal transmission.

VTU – 2018

Electrical Conductivity in Metals and Semiconductors

Mobility of charge carriers...

5.1 Review of Drude – Lorentz theory or Classical free electron theory

Drude proposed his theory of electrical and thermal conduction in metals by applying the highly successful kinetic theory of gasses to a metal, considered and imagined to be composed of gas of electrons. In its simplest form, kinetic theory treats metals are usually crystalline solids. In most cases, they have a relatively simple crystal structure distinguished by a close packing of atoms and a high degree of symmetry. Typically, the atoms of metals contain less than half the full complement of electrons in their outermost shell. Because of this characteristic, metals tend not to form compounds with each other.

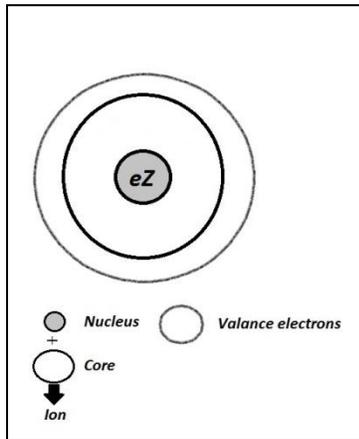


Fig. 5.1: Single isolated atom (not to scale)

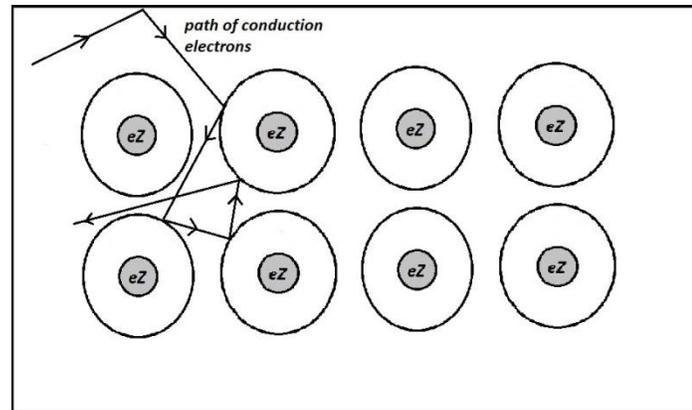


Fig. 5.2: free moment of electrons in the vicinity of ionic lattice

In metallic atoms the outermost shells has valance electrons and are loosely bound to the nuclei. In a metal these atoms are as close together as shown in the figure 5.2. The electrons are tends to move freely in the vicinity of ionic lattice within the body of the metal. This leads to the homogeneous distribution of negative charge (electrons) in the crystal lattice. The concentration of the electrons is almost constant throughout the lattice except at the site of nuclei. The electrons moving between the ions balance the force of repulsion between the positive ions. Hence a stable lattice is formed.

The metallic bond formed between all the atoms of the crystal leads to the collectivization of the electrons. These collectivized electrons move freely in the lattice called *conduction electrons*. The average distance travelled by the conduction electrons between two lattice ions is called *mean free path* (λ) and the average time take for successive transition between the ions is called the *mean collision time* (τ). Hence, they are no longer localized near their parent nuclei. These conduction electrons in a metal contribute to the electrical and thermal conductivity of the metal. Drude compared the free electrons in a metal as that of the molecules in a gas and applied kinetic theory of gasses to these conduction electrons of mass m_e . As the molecules in a gas moves randomly, the electrons in the metal is also treated as randomly moving electrons, In spite of strong electron – electron and electron – ion electromagnetic interactions. The Drude model boldly treats the electrons as gas by the methods of kinetic theory of a neutral dilute gas, with slightly modifications. By laws of kinetic theory of gasses, the rms velocity or the thermal velocity of randomly moving electrons is given by

$$v_{th} = \sqrt{3kT/m}$$

For a conductor under the influence of electric field “*There is a net displacement of electrons in direction opposite to the electric field E called **drift velocity** of electrons*”

Under the steady state conditions, the drift velocity of electrons in a conductor is given by

$$|v_d| = \frac{eE\tau}{m}$$

Due to the drift velocity of the electrons in a conductor, there is an electric current in conductor

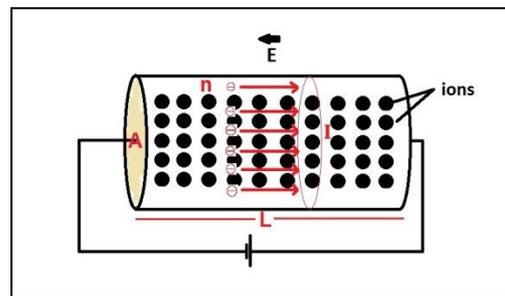
Consider a conductor of length ‘ L ’ and area of cross-section ‘ A ’. In the presence of electric field ‘ E ’, the ‘ n ’ number of electrons drift with the velocity ‘ v_d ’ along the length of the conductor, constitutes to electric current ‘ I ’.

$$I = neAv_d$$

Then, the current density J is given by

$$J = nev_d$$

It is a vector quantity, parallel to the flow of charges, whose magnitude is the amount of charge per unit time crossing a unit area perpendicular to the flow.



Then, the electrical conductivity in metals is given by

$$\sigma = \frac{J}{E}$$

$$\sigma = \frac{I}{AE}$$

$$\sigma = \frac{ne^2\tau}{m}$$

The Resistivity of the metal is given by $\rho = \frac{m}{ne^2\tau}$

5.2 The failure of classical free electron theory

Classical free electron theory fails to explain the following factors

- i. Specific heat of a solid (C_V)
 - ii. Dependence of electrical conductivity (σ) on temperature (T)
 - iii. Dependence of electrical conductivity (σ) on electron concentration (n)
- i. **Specific heat of a solid (C_V):** The heat capacity per mole of conduction electrons depends on the average energy of electrons in three degrees of freedom, under thermal equilibrium and is given by.

$$\bar{E} = \frac{3}{2} (N_A kT)$$

$$C_V = \frac{\partial \bar{E}}{\partial T} = \frac{3}{2} R$$

Therefore molar specific heat of gas at constant volume is given by

$$C_V = \frac{3}{2} R$$

As per classical free electron theory, the free electrons are treated as gas molecules. Thus the above equation must be applicable for free electrons also. But, the experimental value for specific heat was found to be

$$C_V = 10^{-4} RT$$

Which is far lower than the expected value and also showed a dependence on temperature which is contrary to the theory

ii. Dependence of electrical conductivity on Temperature

It was experimentally observed that for metals, the electrical conductivity is inversely proportional to temperature.

$$\sigma \propto \frac{1}{T}$$

According to classical free electron theory,

$$\sigma = \frac{ne^2\tau}{m}$$

But, $v_{th} = \lambda/\tau$ and also $v_{th} = \sqrt{3kT/m}$

Thus,

$$\sigma = \frac{ne^2\lambda}{\sqrt{3mkT}}$$

This equation suggests that the temperature dependence of conductivity is

$$\sigma \propto \frac{1}{\sqrt{T}}$$

Thus, it is clear that the prediction of classical free electron theory is not agreeing with the experimental observations.

iii. Dependence of electrical conductivity on electron concentration

As per classical free electron theory, the electrical conductivity is ,

$$\sigma = \frac{ne^2\tau}{m}$$

Where 'n' is the electron concentration. From the above equation, we see that

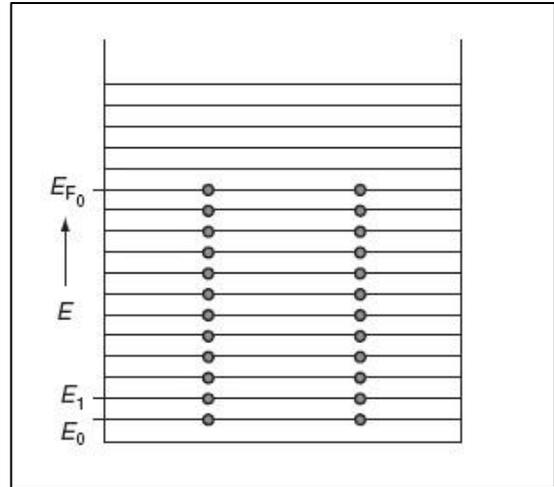
$$\sigma \propto n$$

It clearly says that, as the electron concentration increases. The electrical conductivity of a conductor must increase. But an Experimental observation shows that, the monovalent materials like copper has more conductivity than the divalent and trivalent materials like

Zinc and cadmium. Hence, Classical free electron theory fails to explain the dependence of electrical conductivity on electron concentration 'n'.

5.3 Quantum free electron theory

To explain the discrepancy of classical free electron theory, we must once again turn up to quantum concepts. According to Quantum mechanics, the energy of an electron in a metal is quantized. Therefore electrons will have energy states rather than continuous energy as per classical theory. Electrons in a metal occupy these energy levels. In doing so, they follow very important quantum principle, the *Pouli's exclusion principle*. According to this principle, an energy level can accommodate at most two electrons, one with spin up, and other with spin down. As the distribution of the electrons take place in energy states, this result in the formation of energy band as shown in the figure.



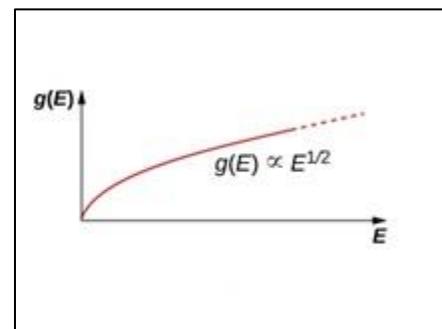
Assumptions of Quantum free electron theory.

- i. The energy of electrons is quantized in a metal and has a set of energy levels.
- ii. The distribution of electrons in the various allowed energy levels occurs as per Pauli's exclusion principle.
- iii. The free electrons travel in a constant potential inside the metal but stay confined within its boundaries.
- iv. The attraction between the free electrons and the lattice ions and the repulsion between the electrons themselves are ignored.
- v. The distribution of energy among the free electrons is according to Fermi-Dirac statistics.

5.4 Density of States in an energy band

It is defined as the number of available states per unit energy range centered at a given energy E in the valence band of a material of unit volume. It is denoted by $g(E)$. It is a continuous function and the product $g(E).dE$ gives the number of states in the energy interval of E and $E+dE$. The Expression is given by

$$g(E) = \frac{8\sqrt{2}\pi m^{3/2}}{h^3} E^{1/2} dE$$



From the above equation we observe that the number of energy levels in an energy states $g(E)$ in interval dE is proportional to $E^{1/2}$. Thus, a plot of $g(E)$ versus E will be always parabolic in nature and as shown in the figure above.

5.5 Fermi- Dirac distribution

Fermi Dirac distribution is a representation which depicts the details of distribution of electrons among the various available energy levels of a material under thermal equilibrium conditions. And it is described by a function called as "*Fermi factor*".

It is defined as "*The probability than an energy sate E is occupied by an electron in an energy band*" given by

$$f(E) = \frac{1}{1 + e^{\left[\frac{E-E_F}{kT}\right]}}$$

Thus, if the level is occupied by electron then, $f(E)=1$ and if not $f(E)=0$. In general $f(E)$ has a value between zero and one.

5.5.1 Dependence of Fermi factor on temperature.

We have Fermi factor

$$f(E) = \frac{1}{1 + e^{\left[\frac{E-E_F}{kT}\right]}}$$

Case (i): When $T=0K$, for $E < E_F$

$$f(E) = \frac{1}{1 + e^{\left[\frac{-ve}{0}\right]}}$$

$$f(E) = \frac{1}{1 + e^{[-\infty]}}$$

$$f(E) = \frac{1}{1 + \frac{1}{e^{[\infty]}}}$$

$$f(E) = \frac{1}{1 + 0}$$

$$f(E) = 1$$

This means that the energy level is certainly occupied and $E < E_F$ applies to all the energy levels below E_F . Thus, at $T = 0 K$, all the energy levels below the Fermi level are occupied.

Case (ii): When $T=0K$, for $E > E_F$

$$f(E) = \frac{1}{1 + e^{\left[\frac{+ve}{0}\right]}}$$

$$f(E) = \frac{1}{1 + \frac{1}{e^{[\infty]}}}$$

$$f(E) = \frac{1}{1 + \infty}$$

$$f(E) = 0$$

Therefore, at $T = 0 \text{ K}$, all the energy levels above the Fermi level are unoccupied. Thus, the variation of $f(E)$ for different energy values becomes a step function at $T = 0 \text{ K}$ as shown.

Case (iii): When $T > 0 \text{ K}$, for $E \cong E_F$

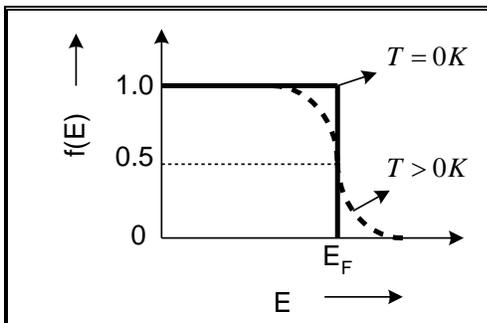
$$f(E) = \frac{1}{1 + e^{\left[\frac{0}{+ve}\right]}}$$

$$f(E) = \frac{1}{1 + e^0}$$

$$f(E) = \frac{1}{1 + 1}$$

$$f(E) = 0.5$$

Thus, at $E = E_F$, the probability of occupation is 0.5. Furthermore, for $E > E_F$, the probability value falls off to '0' rapidly.



5.6 Fermi Energy and Fermi level: In an energy band “The Energy of the highest occupied level at absolute zero temperature is called as Fermi energy E_F or E_{F_0} ”. The corresponding level is called as called as Fermi level.

5.7 Expression for the Fermi Energy (E_F):

Let us consider an energy band for a conductor of unit volume. If ‘n’ be the number of free electrons per unit volume possessing energy only in the range E and $E+dE$. Then, $N(E)dE$ is given by the product of the number of available states in the energy range E and $E+dE$, $g(E)$, and the probability of occupation or the energy states, $f(E)$.

$$N(E)dE = g(E)dE \cdot f(E)$$

The electrons that are distribute in various energy levels up to the Fermi level. Thus, ‘n’ can be evaluated by integrating $N(E)dE$ in the limits $E = 0$ and $E = E_F$.

$$n = \int_0^{E_{F0}} N(E).dE = \int_0^{E_{F0}} f(E).g(E).dE$$

But for all energy levels between $E = 0$ and $E = E_{F0}$, $f(E) = 1$ at $T = 0$ K.

$$n = \int_0^{E_{F0}} g(E).dE$$

$$g(E)dE = \left(\frac{8\sqrt{2}\pi m^{\frac{3}{2}}}{h^3} \right) E^{\frac{1}{2}} dE$$

$$n = \frac{8\sqrt{2}\pi m^{\frac{3}{2}}}{h^3} \int_0^{E_{F0}} E^{\frac{1}{2}} dE = \frac{8\sqrt{2}\pi m^{\frac{3}{2}}}{h^3} \frac{2}{3} E_F^{\frac{3}{2}}$$

Simplifying and rearranging the equation,

$$E_F = \frac{h^2}{8m} \left(\frac{3n}{\pi} \right)^{\frac{2}{3}} = Bn^{\frac{2}{3}}$$

5.8 Merits of Quantum free electron theory.

Quantum free electron theory successfully explains the following factors

- i. Specific heat of a solid (C_V)
 - ii. Dependence of electrical conductivity (σ) on temperature (T)
 - iii. Dependence of electrical conductivity (σ) on electron concentration (n)
- i. Specific heat of a solid (C_V):** According to quantum free-electron theory, only those electrons occupying energy levels close the Fermi energy can absorb the heat energy. Such electrons constitute a very small portion of the total free electrons, thus the specific heat value becomes very small. Further, it was found that

$$C_V = \frac{kT}{E_F} RT = 10^{-4} RT$$

But, the experimental value for specific heat was found to be

$$C_V = 10^{-4} RT$$

This is in agreement of the observed values.

ii. Dependence of electrical conductivity on Temperature

As per quantum free electron theory, the electrical conductivity for a metal is.

$$\sigma = \frac{ne^2\lambda}{mv_F}$$

According to quantum theory theory, E_F and v_F are essentially independent of temperature and λ is inversely proportional to temperature. $\lambda \propto 1/T$

Therefore,

$$\sigma \propto 1/T$$

Thus, dependence of conductivity on temperature is agreed with experimental.

iii. Dependence of electrical conductivity on electron concentration

As per Quantum free electron theory, the electrical conductivity is ,

$$\sigma = \frac{ne^2\lambda}{mv_F}$$

Where 'n' is the electron concentration. From the above equation, we see that

$$\sigma \propto n$$

And also

$$\sigma \propto \frac{\lambda}{v_F}$$

If we compare the case of copper and aluminium, the value of 'n' for Al is 2.13 times higher than that of Cu. But the value of (λ/v_F) for Cu is 3.73 times higher than that for Al. Thus the conductivity of Al is less than that of Cu. This obeys Experimental observations, the monovalent materials like copper has more conductivity than the divalent and trivalent materials like Zinc and aluminium.

6.1 Semiconductors

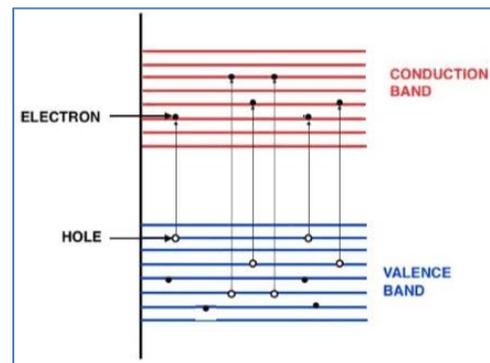
“The materials for which the electrical properties lie between that of conductor and insulator called Semiconductors”. Such as Si, Ge and α – Sn (grey tin) are the class of group IV semiconductors – C (diamond) crystalline structure lie in fourth column of periodic table. Another important group of semiconductors is the Group III – V compounds, third and fifth column in periodic table. Such as the combination of GaAs, InSb, GaP, InAs, GaSb...etc.

The Semiconductors of two types

- i. Intrinsic Semiconductors
- ii. Extrinsic Semiconductors.

- I. **Intrinsic Semiconductors:** These are the pure form of the semiconductors, means without any dopants in a semiconductor.
- II. **Extrinsic Semiconductors:** These are the doped intrinsic semiconductors or impure form of semiconductors.

According to the band theory, there exists a band gap between the valence band and conduction band in semiconductors. At Absolute zero kelvin, the valence band is completely filled and conduction band is completely empty. In general, these are partially empty or partially filled at room temperatures. The forbidden energy gap of these semiconductors is less than 2eV. The electrical conductivity is due to both electrons and holes. During the conductivity the top of the valence band is populated by holes and bottom of the conduction band with electrons in an energy band and as shown in figure for an intrinsic semiconductor.



The expression for the electron concentration in conduction band is evaluated by considering the number of energy states occupied by electrons in the energy range of E to E+dE in CB, this is $g(E) dE$ and their probability of occupancy $f(E)$. The expression for electron concentration is given by.

$$n_e = 2 \left[\frac{2m_e^* kT}{h^2} \right]^{\frac{3}{2}} e^{\left[\frac{E_C - E_F}{kT} \right]}$$

Similarly, the expression for hole concentration in valence band is given by.

$$n_h = 2 \left[\frac{2m_h^* kT}{h^2} \right]^{\frac{3}{2}} e^{\left[\frac{E_F - E_V}{kT} \right]}$$

Where, m_e^* , m_h^* are the effective mass of electron and holes in the presence of electric field

6.2 Fermi level in Intrinsic Semiconductor

For an intrinsic semiconductor, the Fermi level lies exactly in mid-way of forbidden energy gap of energy difference E_g . The concentration of electrons (n_e) and concentration of holes (n_h) are equal. And can be written as,

$$n_e = n_h$$

$$2 \left[\frac{2m_e^* kT}{h^2} \right]^{\frac{3}{2}} e^{\left[\frac{E_c - E_F}{kT} \right]} = 2 \left[\frac{2m_h^* kT}{h^2} \right]^{\frac{3}{2}} e^{\left[\frac{E_F - E_V}{kT} \right]}$$

$$e^{\left[\frac{E_c - E_F - E_F + E_V}{kT} \right]} = \left[\frac{m_h^*}{m_e^*} \right]^{\frac{3}{2}}$$

Taking natural log on both sides, we get

$$\left[\frac{E_c + E_V - 2E_F}{kT} \right] = \ln \left[\frac{m_h^*}{m_e^*} \right]^{\frac{3}{2}}$$

$$E_c + E_V - 2E_F = kT \ln \left[\frac{m_h^*}{m_e^*} \right]^{\frac{3}{2}}$$

$$2E_F = E_c + E_V - kT \ln \left[\frac{m_h^*}{m_e^*} \right]^{\frac{3}{2}}$$

$$E_F = \frac{E_c + E_V}{2} - \frac{kT}{2} \ln \left[\frac{m_h^*}{m_e^*} \right]^{\frac{3}{2}}$$

If $m_e^* = m_h^*$, then above equation becomes,

$$E_F = \frac{E_c + E_V}{2}$$

6.3 Electrical Conductivity in Intrinsic Semiconductors

Consider an intrinsic semiconductor of length 'L' cross-section 'A' and of unit volume 'V'. In the presence of electric field 'E' the electrons moves in the direction opposite to 'E' and hole along its direction.

The drift velocity of electrons is $[v_d]_e = \frac{eE\tau_e}{m_e^*}$

The drift velocity of holes is $[v_d]_h = \frac{eE\tau_h}{m_h^*}$

The current due to this drift velocity is $I = I_e + I_h$

But, In general, the equation for electric Current is $I = n e A v_d$

Using the same analogy, we get electric current in a semiconductor.

$$I = n_e e A [v_d]_e + n_h e A [v_d]_h$$

The current density is

$$J = \frac{I}{A}$$

On substituting for 'I', we get

$$J = n_e e [v_d]_e + n_h e [v_d]_h$$

By, Ohms law The electrical conductivity is $\sigma = \frac{J}{E}$

$$\sigma = n_e e \frac{[v_d]_e}{E} + n_h e \frac{[v_d]_h}{E}$$

But, the mobility is $\mu = \frac{v_d}{E}$

Therefore The electrical conductivity is

$$\sigma = n_e e \mu_e + n_h e \mu_h$$

As, $n_e = n_h = n_i$

$$\sigma = n_i e (\mu_e + \mu_h)$$

6.4 Hall Effect:

When current carrying semiconductor or a conductor is placed in an external uniform magnetic field 'B' at right angles, an electric field 'E' is produced perpendicular to both velocity vector 'v' and magnetic field. This is called the Hall Effect.

6.5 Hall Coefficient:

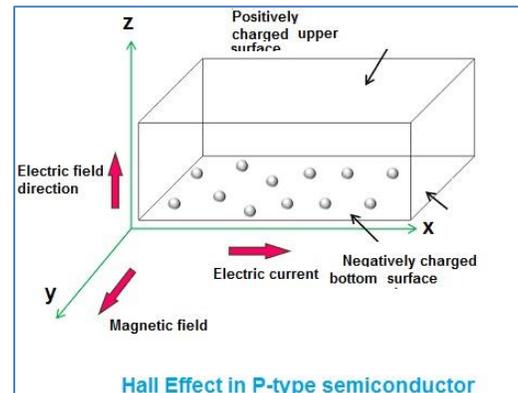
Consider a P - Type semiconductor of uniform cross section 'A', a potential difference of 'V' is applied across the ends, hence the drift velocity of majority charge carriers are holes move in positive x - direction and electrons in negative x - direction. A uniform magnetic field 'B' is applied along the y - direction results in an induced electric field in z - direction. Therefore the upper surface is positively charged and the bottom surface is negatively charged.

The electric force on charges is

$$F_E = eE$$

The magnetic force is

$$F_B = ev_d B_H$$



Where v_d is drift velocity and B_H is the hall field

Under steady state conditions, these are equal.

$$hE = hv_d B_H$$

$$v_d = \frac{E}{B_H}$$

The current density on the surfaces is

$$J = nhv_d$$

On substituting for drift velocity, we get

$$J = nh \frac{E}{B_H}$$

Therefore the Hall coefficient is

$$R_H = \frac{1}{nh} = \frac{E}{JB_H}$$

For negatively charged electrons, Hall coefficient is given by

$$R_H = \frac{-1}{ne} = \frac{E}{JB_H}$$

Reference:

1. Solid state Physics by Ashcroft and Mermin
2. Elementary Solid State Physics by M Ali Omar
3. Solid State Physics by S O Pillai
4. NPTL sources

Engineering Physics

Dielectric materials

$$U = \frac{1}{2} QV$$

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1.1 Dielectric materials

The insulator materials are non-conductive in nature like paper, quartz, mica...etc. But a class of insulators has the property of inducing the charges under the application of electric field 'E'. Hence, find the application in storing the electric charge devices, Such as capacitors and many more.

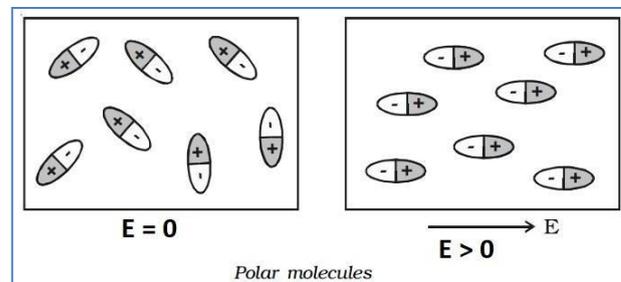
The dielectric materials can be defined as *"The material has the ability to get electrically polarised in the application of electric field"*.

There are two types of dielectric materials.

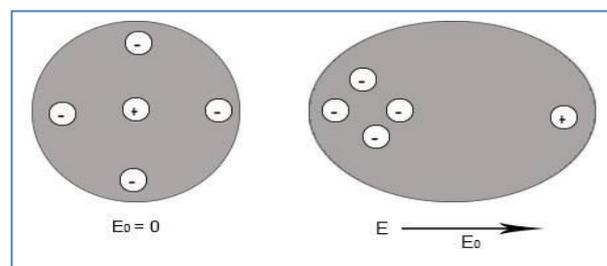
- i. Polar dielectrics
- ii. Non-polar dielectrics

- i. **Polar Dielectrics:** The effective centre of positive charges does not coincide with the effective centre of negative charges like NaCl, H₂O etc. Such materials are called as polar dielectrics. These materials do have permanent dipoles. In which positive and negative charges are well separated by a small distance.

When these materials placed under the influence of electric field, the permanent dipoles undergoes torque $r_{ext} = \mu E \sin\theta$ this makes the dipoles to align along the direction of applied field 'E'. The dipoles in the dielectrics, as shown in the figure.



- ii. **Non - Polar Dielectrics:** In these materials the effective centre of positive charges coincides with the effective centre of negative charges like H₂, CO₂, etc. called non - polar dielectrics. These materials do not have



permanent dipoles, and atomic in nature. When the electric is zero (not applied $E = 0$) the the positive charge is at the center of the atomic structure and surrounded by negative charged electrons. If electric field is applied ($E > 0$). The positive charge moves along the direction of the field and negative charges move oppositely, creating a dipole, as shown in the figure.

In both polar and non-polar dielectrics, if 'a' is the distance between positive and negative charges. Then, dipole momentum (μ) is *"the product of magnitude of the either charge 'q' and the separation 'a' between them"*. It is given by

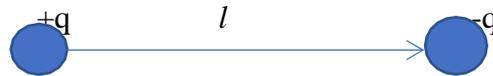
$$\mu = |q|a$$

1.2 Internal Field in the dielectrics

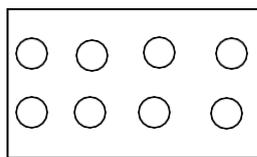
When a solid or liquid dielectric material is placed in external magnetic field lines, each of the atom or molecule develops a dipole and has dipole moment due to electric force. Therefore, *"The resultant field at any given dipole will be the sum*

of applied electric field, and electric field due to surrounding dipoles called as internal field within the dielectric”.

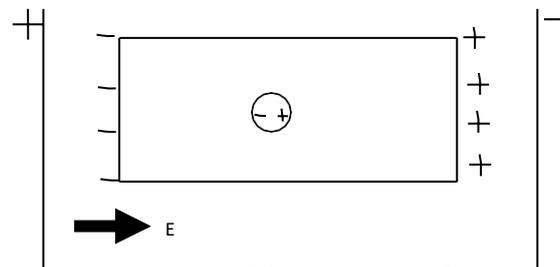
Dipole: Two equal and opposite charges separated by a small distance constitutes a dipole.



Polarization: The separation of effective center of positive and negative charges in a substance by the application of electric field is known as polarization



Atoms in a dielectric in the absence of electric field



Atom in a dielectric in presence of electric field

Atomic polarizability $p = \alpha E$ where α is the polarization constant.

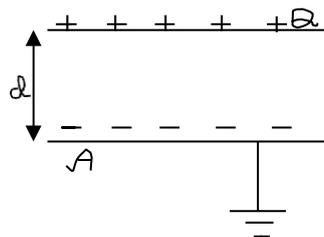
Polarization P= Dipole Moment per unit volume $= \epsilon_0 E(\epsilon_r - 1) = N\alpha E_i$

1.3 EXPRESSION FOR STATIC DIELECTRIC CONSTANT:

Consider a parallel plate capacitor of area A, Charge density σ and total charge Q.

From Gauss's law, Electric intensity between the plates $E = \frac{\sigma}{\epsilon_0 \epsilon_r}$

Potential difference $V = E \cdot d = \frac{\sigma}{\epsilon_0 \epsilon_r} \cdot d$



(a) In the absence of a dielectric

$$\text{Capacitance } C_{\text{without dielectric}} = \frac{Q}{V} = \frac{\sigma A}{\frac{\sigma d}{\epsilon_0}} = \epsilon_0 \frac{A}{d}$$

(a) In the presence of a dielectric $C_{\text{with dielectric}} = \frac{\epsilon_0 \epsilon_r A}{d}$

$$\text{Static dielectric constant } \epsilon_r = \frac{C_{\text{with dielectric}}}{C_{\text{without dielectric}}}$$

Dielectric constant is defined as the ratio of capacitance of a capacitor with a dielectric to its capacitance in the absence of a dielectric.

1.4 DIFFERENT POLARIZATION MECHANISMS:

When a dielectric material is subjected to external electric field, the electrical polarization of the material occurs by any one of the following mechanisms. There are 4 mechanisms namely,

1. Electronic polarization
2. Ionic polarization
3. Orientation polarization
4. Space charge polarization

Electronic polarization: These are generally seen in the case of covalent compounds.

When a covalent compound is placed in electric field, displacement of electron cloud takes place relative to the nucleus. This displacement creates a dipole which develops dipole moment.

$$\text{Electronic polarizability } \alpha_e = \frac{\epsilon_0 (\epsilon_r - 1)}{N}$$

N is number of dipoles per unit volume

It is independent of temperature.

Electronic polarization set over a period of 10^{-14} to 10^{-15} s

Ionic polarization:

This is exhibited by ionic compounds like NaCl, KCl.

When ionic compounds are kept in an electric field, displacement of positive and negative ions occurs developing a dipole moment.

$$\text{Ionic polarizability } \alpha_i = \frac{\epsilon_0 (\epsilon_r - 1)}{N_i}$$

It is also independent of temperature. The ionic compounds experience Electronic polarisation in addition to it. The ionic polarisation occurs for a period of 10^{-11} to 10^{-14} s.

Orientation polarization:

This type of polarization occurs in polar substances in which there are permanent molecular dipoles but are orientated randomly due to thermal agitation. When polar molecules are kept in an electric field, the already existing dipoles tend to align in the direction of applied electric field. This increases the dipole moment.

$$\text{Orientation polarization } \alpha_o = \frac{\mu^2}{kT}$$

The build up time is in the order of 10^{-10} s. The orientational polarization is strongly temperature dependent.

Higher the temperature, lower will be the polarization.

Space charge polarization:

This polarization exists in materials possessing different phases due to difference in temperatures. In such materials charge carriers drift and accommodate in certain regions of higher conductivity (electrodes) causing dipole moment. It occurs in ferrites and semiconductors. Its magnitude is very small compared to other mechanisms.

1.5 RELATION BETWEEN POLARIZATION AND DIELECTRIC CONSTANT:

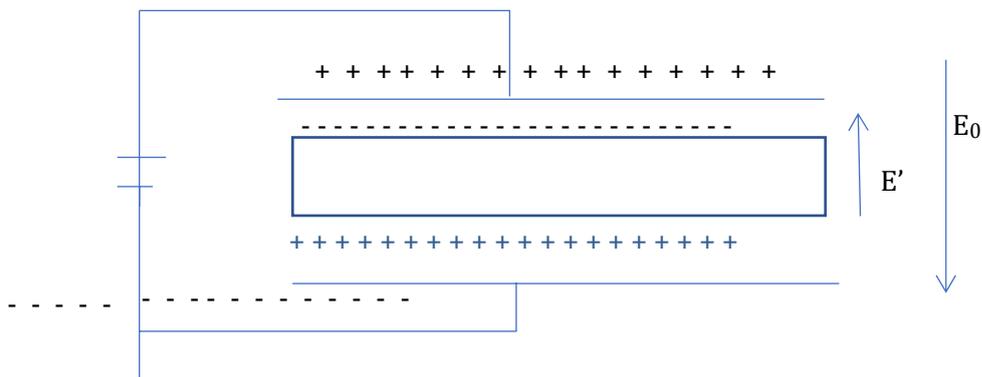
Consider a parallel plate capacitor with plates between which an electric field E_0 exists. If σ is the charge per unit area on the plates, then from Gauss law, we have

$$E_0 = \sigma / \epsilon_0 \quad \longrightarrow \quad (1)$$

Let the given dielectric slab be placed between the two plates. Due to polarization, charges appear at the faces of the dielectric slab and establish another field E' within the dielectric which is opposite to the applied field's direction. The resultant field is given by

$$E = E_0 - E' \quad \longrightarrow \quad (2)$$

Let σ_p be the charge per unit area on the surface of the dielectric, then $E' = \sigma_p / \epsilon_0 \quad \longrightarrow \quad (3)$



Therefore eqn (2) becomes $E = \sigma / \epsilon_0 - \sigma_p / \epsilon_0$

$$E \epsilon_0 = \sigma - \sigma_p \quad \longrightarrow \quad (4)$$

Since, Polarization=Induced charges per unit area, $P = \sigma_p$ and by Gauss law $D = \sigma$ where D is called electric flux density.

Therefore, $E \epsilon_0 = D - P$

$$P = D - E \epsilon_0$$

From Electrostatics,

$$D = \epsilon_0 \epsilon_r P = \epsilon_0$$

$$\epsilon_r E - \epsilon_0 E$$

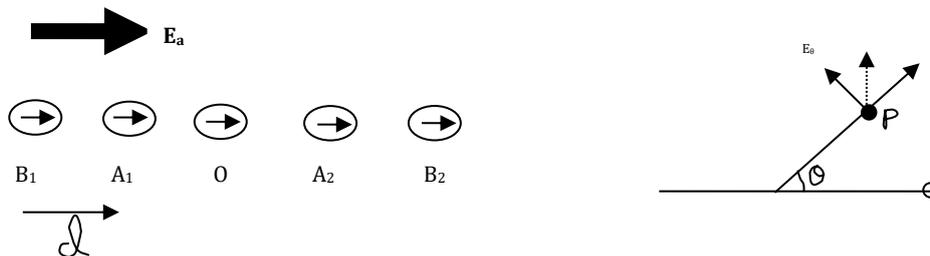
$$P = \epsilon_0 (\epsilon_r - 1)E$$

$$P = \epsilon_0 \chi E$$

where χ is the dielectric susceptibility of the material.

1.6 INTERNAL FIELDS IN A DIELECTRIC:

Internal field or local field is the resultant of the applied field and the field produced due to all the dipoles. Consider a dielectric material kept in an electric field E_a .



The dipole is assumed to be in a one-dimensional array and are oriented in the same direction. In that array, let us consider an atom O which is at a distance 'd' from A_1 and '2d' from B_1 and so on.

The electric field at O due to dipole A_1 is given by

$$E_{A1} = \frac{\mu \cos \theta}{2\pi \epsilon d^3} + \frac{\mu \sin \theta}{4\pi \epsilon d^3} = \frac{\mu}{2\pi \epsilon d^3} \text{ as } \theta = 0$$

The electric field at O due to A_2

$$E_{A2} = \frac{\mu}{2\pi \epsilon d^3} \text{ as all dipoles are oriented in the same direction.}$$

Field at O due to A_1 and A_2 is $E_{A1, A2} = \frac{\mu}{\pi \epsilon d^3} = E_1$

Similarly field at O due to B_1 and B_2 is $= \frac{\mu}{\pi \epsilon (2d)^3} = E_2$

The resultant field due n dipoles is given by

$$E_R = E_1 + E_2 + E_3 + E_4 + E_5 + E_6 + \dots$$

$$= \frac{\mu}{\pi \epsilon d^3} + \frac{\mu}{\pi \epsilon (2d)^3} + \dots$$

The internal electric field is $E_i = E_a + \frac{1.2\mu}{\pi \epsilon d^3}$

$$E_i = E_a + \frac{1.2\mu}{\pi \epsilon d^3} \quad \because \mu = \alpha E_a$$

In three dimensional case, $(1/d^3)$ could be replaced by N , the number of atoms per unit volume and $(1.2/\pi)$ by a constant γ which depends on the crystal structure.

Hence
$$E_i = E_a + \frac{\gamma N \mu}{\epsilon} = E + \left[\frac{\gamma}{\epsilon_0} \right] P \quad \because N \mu = P$$

1.7 CLAUSIUS – MOSOTTI RELATION:

This expression relates dielectric constant of an insulator (ϵ) to the polarization of individual atoms (α) comprising it.

$$\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{N\alpha}{3\epsilon_0}$$

where N is the number of atoms per unit volume

α is the polarisability of the atom

ϵ_r is the relative permittivity of the medium

ϵ_0 is the permittivity of free space.

Proof:

If there are N atoms per unit volume, the electric dipole moment per unit volume –known as polarization is given by

$$P = N\alpha E$$

By the definition of polarization P , it can be shown that

$$P = \epsilon_0 E (\epsilon_r - 1) = N\alpha E_i$$

$$\epsilon_0 \epsilon_r E - \epsilon_0 E = N\alpha E_i$$

$$\epsilon_r = 1 + \frac{N\alpha E_i}{\epsilon_0 E} \dots\dots\dots(1)$$

The internal field at an atom in a cubic structure ($\gamma = 1/3$) is of the form

$$E_i = E + \frac{p}{3\epsilon_0} = E + \frac{N\alpha E_i}{3\epsilon_0}$$

$$\frac{E_i}{E} = \frac{1}{\left[1 - \left(\frac{N\alpha}{3\epsilon_0}\right)\right]}$$

Substituting for $\frac{E_i}{E}$ in equation (1)

$$\epsilon_r = 1 + \frac{N\alpha}{\epsilon_0} \left[\frac{1}{\left(1 - \frac{N\alpha}{3\epsilon_0}\right)} \right] = \frac{\epsilon_0 \left[1 - \frac{N\alpha}{3\epsilon_0}\right] + \frac{N\alpha\epsilon_0}{\epsilon_0}}{\epsilon_0 \left[1 - \frac{N\alpha}{3\epsilon_0}\right]} = \frac{1 + \frac{2}{3} \left(\frac{N\alpha}{\epsilon_0}\right)}{1 - \frac{1}{3} \left[\frac{N\alpha}{\epsilon_0}\right]}$$

$$\left[\frac{\epsilon_r - 1}{\epsilon_r + 2} \right] = \frac{\frac{1 + (2/3) \frac{N\alpha}{\epsilon_0}}{\epsilon_0} - 1}{1 - (1/3) \frac{N\alpha}{\epsilon_0}} = \frac{N\alpha}{3\epsilon_0}$$

$$\frac{\epsilon_r - 1}{1 - (1/3) \frac{N\alpha}{\epsilon_0} + 2} = \frac{N\alpha}{\epsilon_0}$$



Course Material

MODULE-5

Material Characterization Techniques and Instrumentation:

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Richard Feynman : “There's Plenty of Room at the Bottom: An Invitation to Enter a New Field of Physics” – Nano science.

Material Characterization and Instrumentation Techniques:

5.Introduction to materials:

5.1 Nanomaterials and Nanocomposites

Nano science and technology is a broad and interdisciplinary area of research and development activity that has been growing explosively worldwide in the past few years. It has the potential for revolutionizing the ways in which materials and products are created and the range and nature of functionalities that can be accessed.

Nano Science is the study and understanding of properties of Nano Particles. Nanotechnology can be defined as the design, characterization, production and application of structures devices and systems by controlling shape and size at a Nano meter Scale.

Nanomaterials are considered as the bridge between bulk materials and atomic or molecular structures. The particles with at least one dimension in the range of 1 to 100 nanometres are called nanoparticles. A nanometre is one millionth of a millimetre - approximately 1,00,000 times smaller than the diameter of a human hair. Nanoparticles have advantages of their small size and high surface to volume ratio. The novel properties that differentiate the nanoparticles from bulk materials are developed at a critical length of 100nm. As the size of the particle reduces to nanometre range ($1 \text{ nm} = 10^{-9} \text{ m}$) the surface to volume ratio increases, this increase in surface to volume ratio, in turn increases the number of atoms on the surface and makes the behaviour of atoms on the surface dominant over those in the interior of the particle.

Nanomaterials are of interest because at this scale unique optical, magnetic, electrical, and other properties emerge. These emergent properties have the potential for great impacts in electronics, medicine, and other fields.

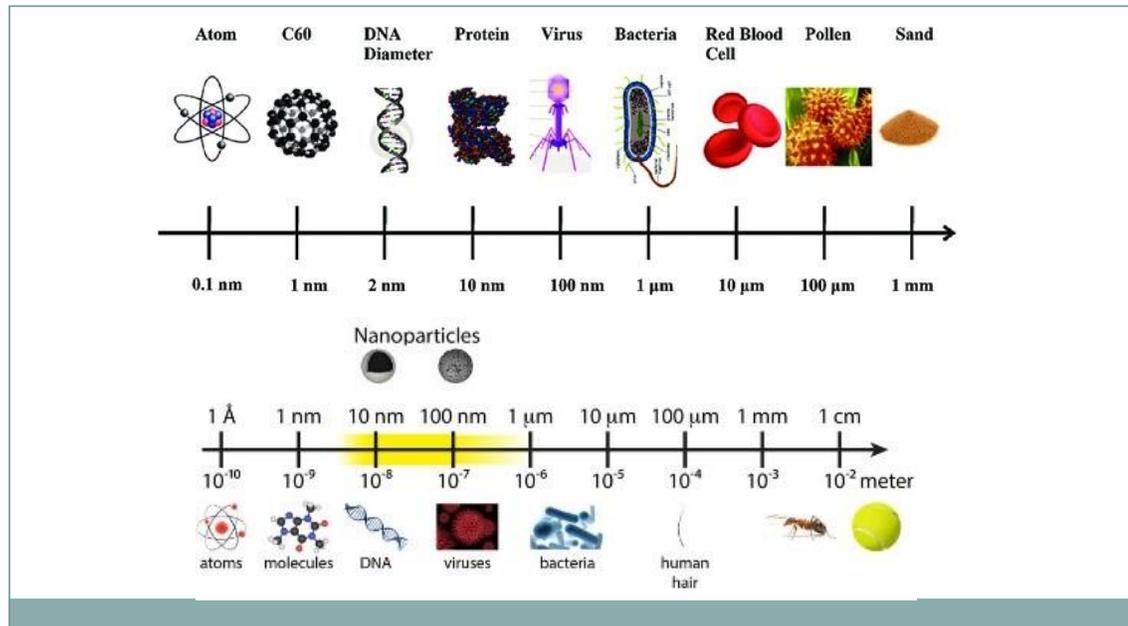


Figure 1: Illustration of size of materials exist in around us

5.2 Special features of Nano phase materials:

The material properties - physical, chemical, optical, thermal, mechanical, electronic, and electrical depend on how their atoms and molecules interact each other and with the neighbouring atoms and molecules. These interactions in turn depend on quantity of the material. In bulk state most of the atoms are inside and interact only with their own kind. But at nanoscale, much higher percentage of atoms are exposed on the surface than in the bulk state. These exposed atoms are thus available to interact with surrounding atoms, for these reasons the shift from bulk to nanoscale can have major impact on properties. These changes lead to the creation of novel materials. Materials that are insulators may become semiconductor or conductors when reduced to nano scale, bulk form of gold is a shiny golden coloured, non-reactive metal. But nanoscale gold will take on different colours from red to green and chemically reactive.

5.3 Nanocomposites

A composite is the combination of two or more individual materials like metals, ceramics, or polymers, to give unique properties in one material. The design purpose of the composite is to achieve best combination of properties superior to those of the individual components and not displayed by any single constituent material. Nanotechnology has major role in developing new

type of materials with improved properties and wide applications. Nanocomposites are one such type of materials and form an exciting field of research in both science and Technology. Novel materials can be obtained from the combination of the best features of both the constituents in a single material. They are expected to exhibit unique properties that emerges from the synergic effect of their components.

A nanocomposite is a compound in which nanomaterials are added into a matrix (base material) to improve a particular property of the material. It can be considered as a multiphase material in which one of the phases has one, two or three dimensions with less than 100nm. The phase can be organic, inorganic or both with distinct differences in structure, chemistry, and properties. Nanocomposites differ from the conventional composites due to high surface to volume ratio of the reinforcing nano phase.

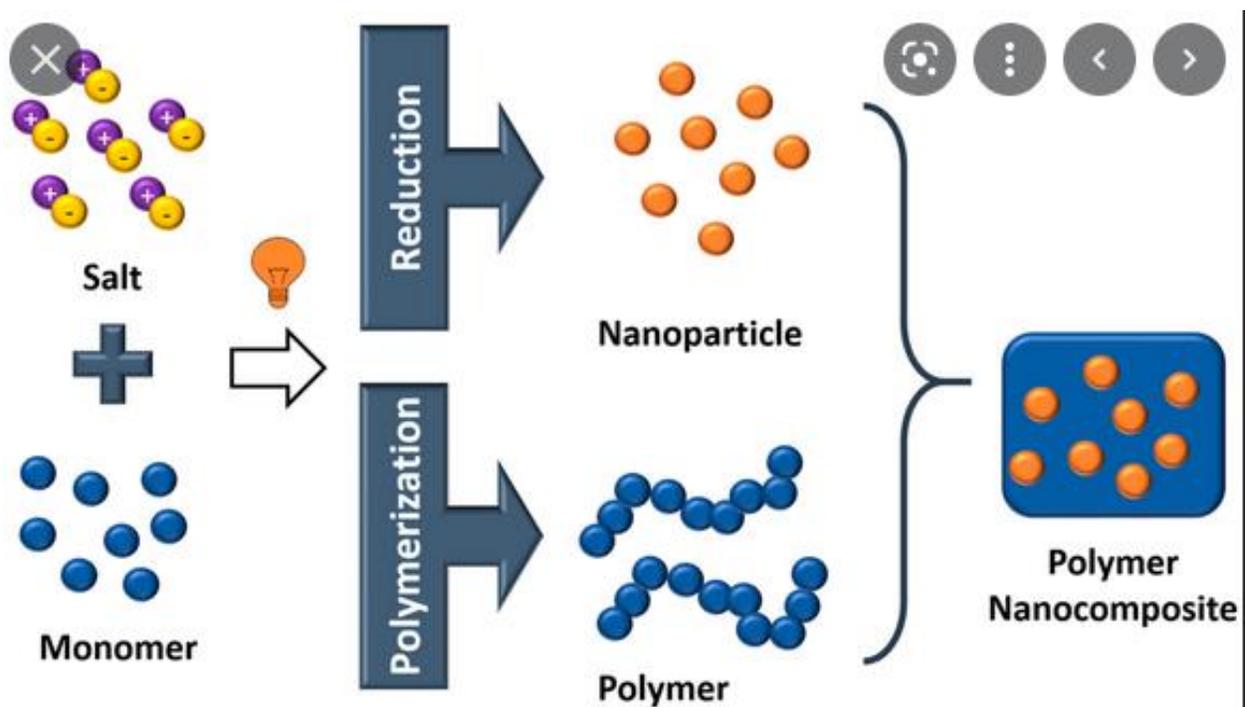


Figure 2: Formation of polymer- nanocomposites

5.4 Classification of nanomaterials:

Nanostructured materials are classified as Zero dimensional, one dimensional, two dimensional, three-dimensional nanostructures.

In *zero-dimensional* (0D) nanomaterials all the dimensions are measured within the nanoscale (no dimensions are larger than 100 nm). Most commonly, 0D nanomaterials are nanoparticles, quantum dots.

In *one-dimensional* nanomaterials (1D), one dimension is outside the nanoscale. This class includes nanotubes, nanorods, and nanowires.

In *two-dimensional* nanomaterials (2D), two dimensions are outside the nanoscale. This class exhibits plate-like shapes and includes graphene, nanofilms, nanolayers, and nanocoating.

Three-dimensional nanomaterials (3D) are materials that are not confined to the nanoscale in any dimension. This class can contain bulk powders, dispersions of nanoparticles, bundles of nanowires, and nanotubes as well as multi-nanolayers.

Two dimensional nanomaterials such as tubes and wires have generated considerable interest among the scientific community in recent years.

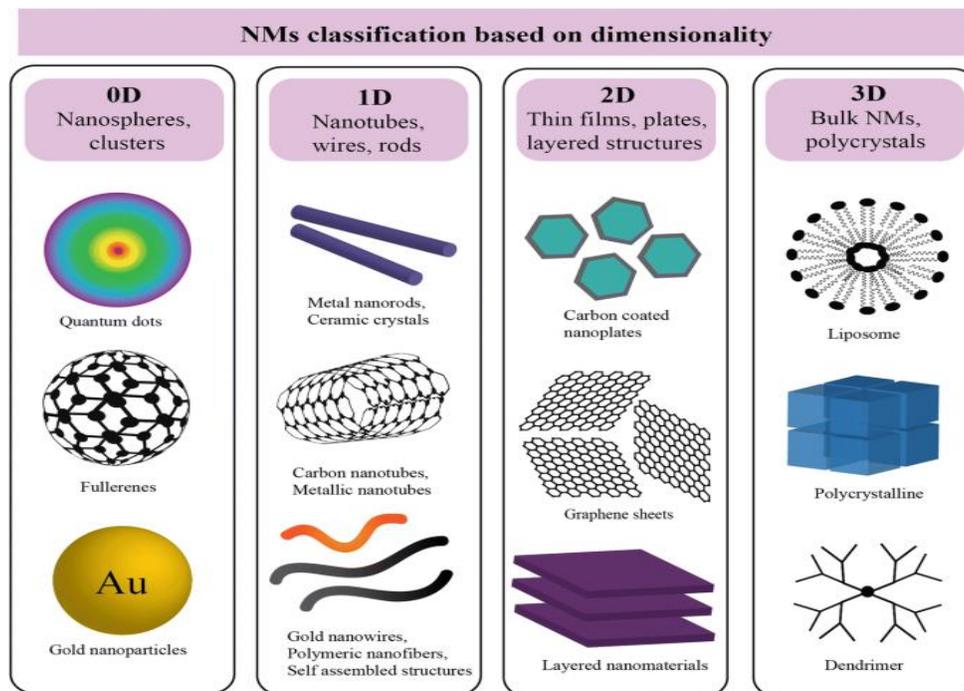


Figure3: Classification of nanomaterials based on dimensions

5.5 X-Ray Diffraction technique

XRD is an effective method for determining the crystal structure of materials. It detects crystalline materials having crystal domains greater than 3-5 nm. It is used to characterize bulk crystal structure and chemical phase composition.

5.5.1. Application:

XRD is a non-destructive technique. Some of the uses of x-ray diffraction are;

- Differentiation between crystalline and amorphous materials;
- Determination of the structure of crystalline materials
- Determination of electron distribution within the atoms, and throughout the unit cell
- Determination of the orientation of single crystals
- Determination of the texture of polygrained materials
- Measurement of strain and small grain size.

Advantages of using X-ray:

X-ray is the cheapest, the most convenient and widely' used method. X-rays are not absorbed very much by air, so the' specimen need not be in an evacuated chamber.

Disadvantage of using X-ray:

They do not interact very strongly with lighter elements.

5.5.2. Principle of X-Ray Diffraction technique

- X-ray diffraction is based on constructive interference of monochromatic X-rays and a crystalline sample.
- These X-rays are generated by a cathode ray tube, filtered to produce monochromatic radiation, collimated to concentrate, and directed toward the sample.
- The interaction of the incident rays with the sample produces constructive interference (and a diffracted ray) when conditions satisfy Bragg's Law ($n\lambda=2d \sin \theta$). This law relates the wavelength of electromagnetic radiation to the diffraction angle and the lattice spacing in a crystalline sample.
- These diffracted X-rays are then detected, processed, and counted. By scanning the sample through a range of 2θ angles, all possible diffraction directions of the lattice should be attained due to the random orientation of the powdered material.

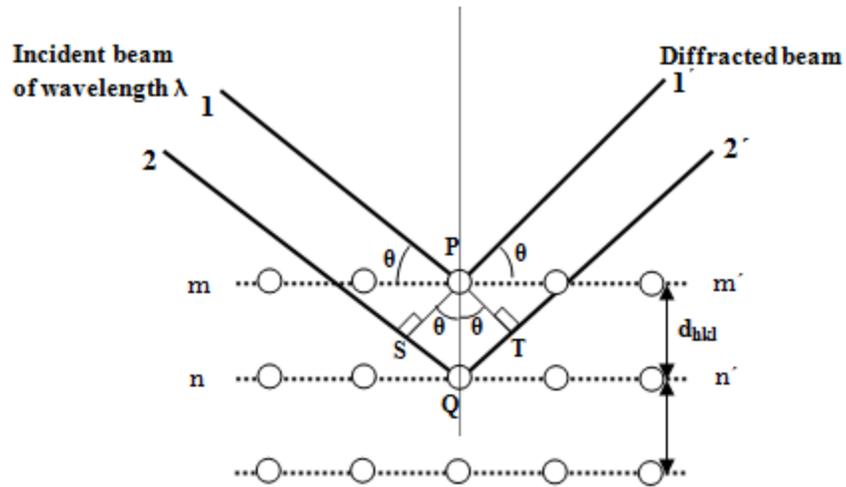


Fig. 5. Bragg’s analysis for X-ray diffraction planes

5.5.3. Construction

In X-ray diffractometer, an X-ray beam of a single wavelength is used to examine the specimens. By continuously changing the incident angle of the X-ray beam, a spectrum of diffraction intensity versus the angle between incident and diffraction beam is recorded.

The main components of diffractometer are

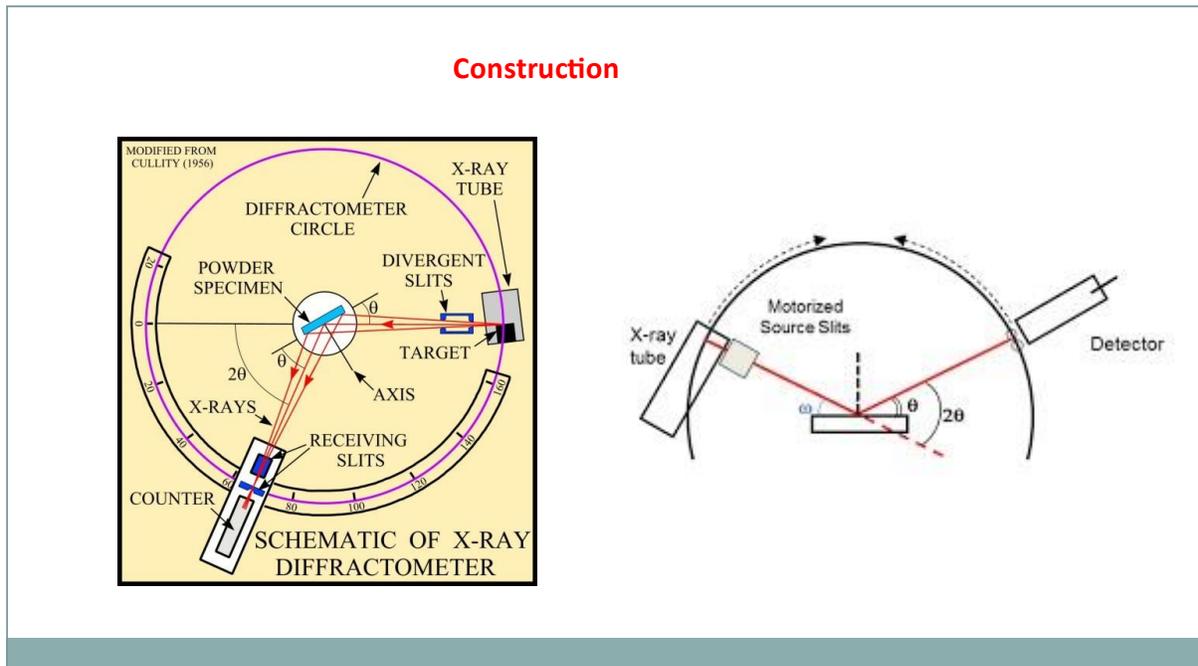


Figure .4: Basic components of X-ray diffractometers

X-ray diffractometers consist of three basic elements: an X-ray tube, a sample holder, and an X-ray detector.

1. X-ray Tube: X-rays are generated in a cathode ray tube by heating a filament to produce electrons, accelerating the electrons toward a target by applying a voltage, and bombarding the target material with electrons. When electrons have sufficient energy to dislodge inner shell electrons of the target material, characteristic X-ray spectra are produced.

2. Sample Holder: It is used to keep the sample in the form of powder so that an X-ray beam will fall into it. The sample holder is attached to the rotating table and the position of the table can be read by means of the Vernier scale.

3. X-ray detector: The X-ray detector is attached to the ionization chamber, which is mounted on a mechanical arm issue, which can turn coaxially with the turntable. However, the mechanical arm is so coupled to the turntable that, for every rotation by an angle θ of the turntable, the mechanical arms turned through 2θ .

5.5.4. Working:

- The sample having many crystallites randomly orientated in all possible orientations is mounted over the sample holder and the collimated beam of the monochromatic X-ray is incident on it with an angle Θ .
- The X-ray beam gets diffracted through the crystallographic plane when the condition of Bragg's law is satisfied, which is expressed as,

$$2 d_{hkl} \sin\Theta = n\lambda$$

where d_{hkl} is the interplanar distance, λ is the wavelength of the X-ray and n is the integer and Θ is the angle called Bragg's angle. The diffracted beam of the X-ray is further detected and counted by the detector with an angle of 2Θ .

- After that, the detector records and processes this X-ray signal and converts the signal to a count rate which is then output to a device such as a printer or a computer monitor.
- The geometry of an X-ray diffractometer is such that the sample rotates in the path of the collimated X-ray beam at an angle θ while the X-ray detector is mounted on an arm to collect the diffracted X-rays and rotates at an angle of 2θ .
- The instrument used to maintain the angle and rotate the sample is termed a goniometer.

5.5.5. XRD pattern of oxide supports:

Titania and zirconia are used as catalyst support or as active catalyst itself. They can be identified from their XRD pattern by comparison with the standard files. For any unknown sample the peaks are identified by comparison with the standard data. For oxides calcined at 400-500°C, the typical XRD profiles of Titania and tetragonal zirconia are shown in Figure 5.

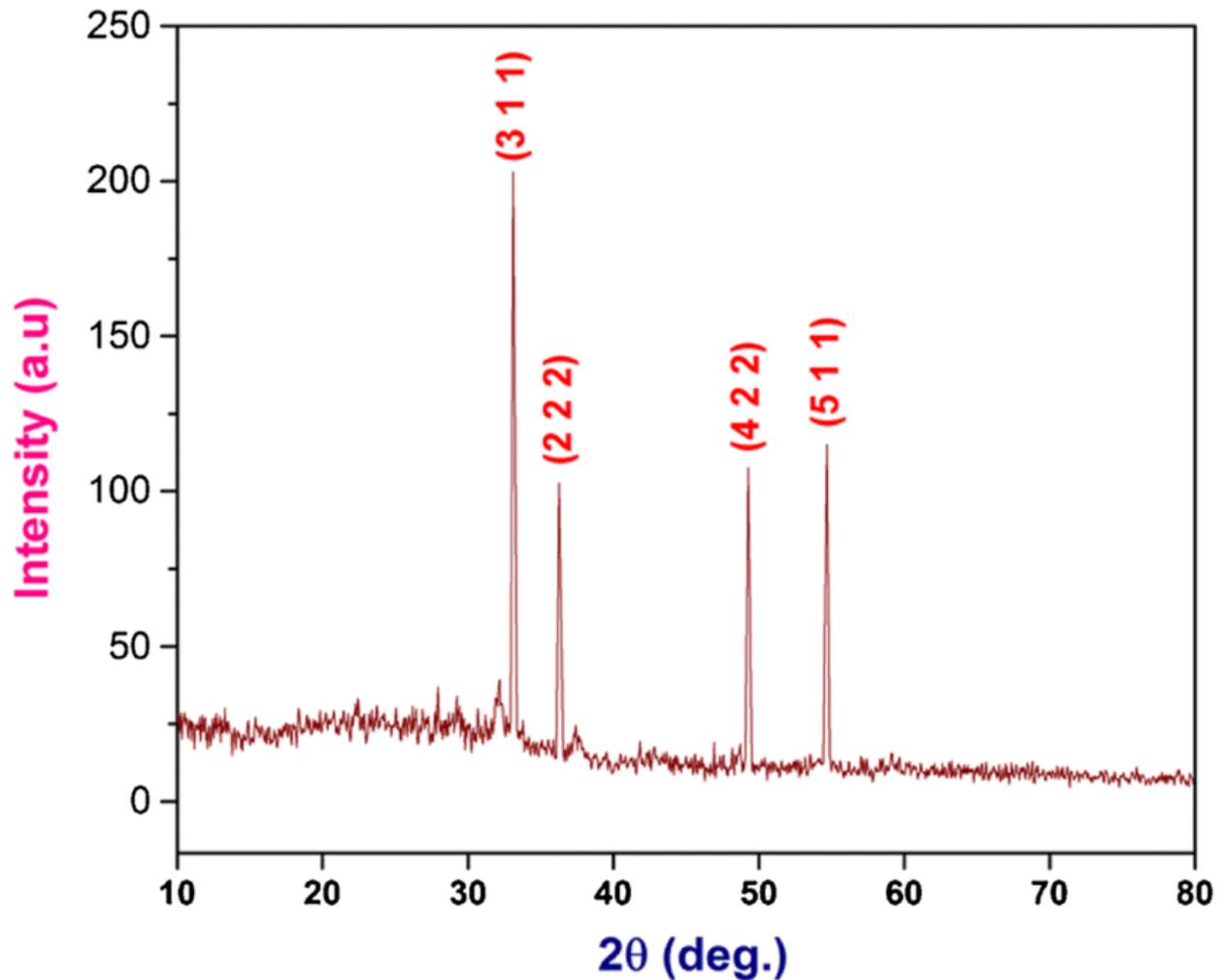


Figure.5. XRD profiles for titania and zirconia prepared by precipitation method

Solved problem

1. A cubic crystal was placed in an x-ray diffractometer using incoming x-rays with a wavelength $\lambda=0.154$ nm. The various peak intensities recorded at different 2θ values are given in following table.

2θ (deg)	40.3	58.3	73.2	154.2	131.2
h k l	110	200	211	400	321
Relative intensity	100	15	23	2	18

Determine the followings

- i. Calculate the crystal size (Given, $k=0.9$; $\beta = 0.0098$)
- ii. Calculate the value for the lattice constant (Assume first order diffraction with $n = 1$.)

Solution

- i. The crystal size can be calculated using the peak corresponding to $2\theta = 40.3$ having highest peak intensity.

Hence, $k = 0.9$; $\beta = 0.0098$; $\lambda=0.154$ nm; $\theta=20.15$, the crystal size L can determine using the Scherrer equation as follows

$$L = \frac{k\lambda}{\beta \cos\theta} = \frac{0.9 \times 0.154}{0.0098 \times \cos(20.15)} = \frac{0.1386}{0.0098 \times 0.9387} = 15.1 \text{ nm}$$

- ii. The plane spacing of cubic crystal is related to the lattice parameter (a) by the following equation.

$$d_{hkl}^2 = \frac{a^2}{h^2 + k^2 + l^2}$$

5.6 DETERMINATION OF CRYSTAL STRUCTURE:

The Scherrer equation, in X-ray diffraction and crystallography, is a formula that relates the size of sub-micrometre crystallites in a solid to the broadening of a peak in a diffraction pattern. It is named after Paul Scherrer. It is used in the determination of size of crystals in the form of powder. The Scherrer equation can be

written as:
$$D = \frac{K\lambda}{\beta \cos\theta}$$

K is a dimensionless **shape factor**, with a value close to unity. The shape factor has a typical value of about 0.9, but varies with the actual shape of the crystallite.

$$D = \frac{0.9\lambda}{\beta \cos\theta}$$

$$D = \frac{0.9\lambda}{\beta \cos\theta}$$

where ' λ ' is the wavelength of the X-rays and ' β ' is the full width at half maximum intensity in radians. ' θ ' is the half of the Bragg angle. The lattice parameters can be calculated from equation using the (hkl) parameters and the interplanar spacing ' d '.

$$d = \frac{a}{\sqrt{h^2+k^2+l^2}}$$

- The Scherrer equation is limited to Nano-scale crystallites, or more-strictly, the coherently scattering domain size, which can be smaller than the crystallite size (due to factors mentioned below). It is not applicable to grains larger than about 0.1 to 0.2 μ m.
- Variety of factors can contribute to the width of a diffraction peak besides instrumental effects and crystallite size; the most important of these are usually inhomogeneous strain and crystal lattice imperfection.
- The Scherrer equation is a widely used tool to determine the crystallite size of polycrystalline samples. However, it is not clear if one can apply it to large crystallite sizes because its derivation is based on the kinematical theory of X-ray diffraction.

- For large and perfect crystals, it is more appropriate to use the dynamical theory of X-ray diffraction. Because of the appearance of polycrystalline materials with a high degree of crystalline perfection and large sizes, the full width at half maximum is then extracted and the crystallite size is computed using the Scherrer equation.

5.6.1. Applications

X-ray diffraction is widely used for structural analysis. Identification of solids is critical in material science, environmental science, and engineering. Other applications include:

1. Material Characterization
2. Determination of dimension of unit cells
3. Identification of materials like clays and mixed layer clays which are difficult to observe optically
4. Assessment of purity of sample
5. Measurement of percentage of crystallinity

5.7 Atomic Force Microscopy (AFM)

The Atomic Force Microscope is a kind of scanning probe microscope in which a topographical image of the sample surface can be achieved based on the interactions between a tip and a sample surface. The atomic force microscope was invented by Gerd Binnig et al. in 1986.

- Atomic Force Microscopy (AFM) is an advanced high-resolution scanning technique used to analyze samples in detail. The technology can magnify images over a hundred million times. The resolution of scans can go up to the Angstrom scale (10^{-10} meters).
- The atomic force microscope (AFM) is a type of scanning probe microscope whose primary roles include measuring properties such as magnetism, height, friction. The resolution is measured in a nanometer, which is much more accurate and effective than the optical diffraction limit. It uses a probe for measuring and collection of data involves touching the surface that has the probe.

5.7.1. Application

The applications of AFM are not limited to a particular field of study. The technology is used for investigating a variety of different samples in the study of physical sciences, life sciences, electronics, and engineering in general.

1. AFM is used to study surface textures, defects, coatings, and tons of other physical features.
2. The technology works great for observing cells and biomolecules in their natural environment.
3. AFM can also be used to analyze microelectronics circuits and components.
4. Energy storage materials like batteries and energy generation materials like photovoltaic cells are studied using AFM.
5. Tribology, surface chemistry, genetic engineering, medicine are other prominent fields where the microscopy technique is used as an important tool for observation and research.

5.7.2. Principle:

The AFM is based on the principle of the interaction between cantilever/tip assembly and the sample; this assembly is also commonly referred to as the probe. The AFM probe interacts with the substrate through a raster scanning motion. During the scanning process, a laser beam is made to fall on the back of the tip, which is coated with a reflective material. The laser beam gets reflected from the shiny surface and falls on a photodetector screen.

5.7.3. Construction:

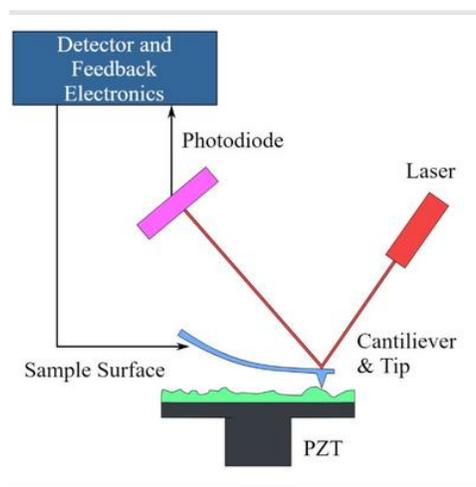


Figure 6: Schematic diagram of construction of AFM

Atomic Force Microscopes have several techniques for measuring force interactions such as van der Waals, thermal, electrical, and magnetic force interactions for these interactions done by the AFM, it has the following parts that assist in controlling its functions.

1. Modified tips are used to detect the sample surface and undergo deflections.
2. Software adjustments used to image the samples.
3. **Feedback loop control** - they control the force interactions and the tip positions using a laser deflector. The laser reflects from the back of the cantilever and the tip and while the tip interacts with the surface of the sample, the laser's position on the photodetector is used in the feedback loop for tracking the surface of the sample and measurement.
4. **Deflection** - The Atomic Force Microscope is constructed with a laser beam deflection system. The laser is reflected from the back of the AFM lever to the sensitive detector. They are made from silicon compounds with a tip radius of about 10 nm.
5. **Force measurement**- the AFM works and depends highly on the force interactions; they contribute to the image produced. The forces are measured by calculation of the deflection lever when the stiffness of the cantilever is known. This calculation is defined by Hooke's law, defined as follows: $F = -kz$, where F is the force, k is the stiffness of the lever, and z is the distance the lever is bent

5.7.4. Working

- The working of AFM is quite straightforward. The sample that needs to be observed is first placed on a stage.
- Then a cantilever with a sharp tip is made to pass over the surface line by line to raster (digital image that uses tiny rectangular pixels) scan the sample in a way.
- During the scanning process, a laser beam is made to fall on the back of the tip, which is coated with a reflective material.
- When the tip encounters bumps or depressions on the surface of the sample, it gets deflected from its original position, causing the laser beam to move too. This movement is detected by a photodetector and sent to a high gain amplifier circuit.

- After adding some gain and converting the signal into a processable entity, the amplifier passes it on to the computer, collating signals received during the entire scan and ultimately providing a 3D profile of the surface. The laser beam gets reflected from the shiny surface and falls on a photodetector screen.

5.8 X-RAY PHOTOELECTRON SPECTROSCOPY (XPS):

5.8.1 INTRODUCTION:

- The technique of X-Ray photoelectron Spectroscopy (XPS) is also known as Electron Spectroscopy for chemical analysis. XPS is a type of electron Spectroscopy it is an analytical technique to study the electronic structure and dynamic in atoms and molecules.
- XPS was invented by Kai SIEGBAHN 1954 and received Nobel Prize in 1981.
- In x-ray photoelectron spectroscopy, primary beam x-ray Photon, which are irradiated on sample surface, the secondary beam (electron) obtained is then analyzed. The Secondary beam is made up of electrons.
- The spectrum of X-Ray photoelectron spectroscopy consists plot of number of electrons or power of electron as a function of energy i.e., kinetic energy or binding energy.

5.8.2 PRINCIPLE:

- Due to the bombardment of X-Ray Photon on the sample surface K and L electron are ejected which are further analysed by the analyser.
- Let us consider E_b , $E_{b'}$ and $E_{b''}$ are binding energy of lower energy levels inner core orbitals. Where E_v , $E_{v'}$ and $E_{v''}$ are the energies of the valence shell electron.
- The monochromatic X-ray Photon incident on the sample surface cell electron abstract the energy from this x-ray Photon and get ejected in terms of electron. Kinetic energy of the ejected electron is recorded by spectrometer and is given by

$$E_k = h\nu - E_b - \phi$$

Where,

E_k is kinetic energy of the ejected electron

$h\nu$ is energy associated with incident Photon

E_b is binding energy of ejected electron

ϕ is the work function of the instrument

5.8.3. Construction:

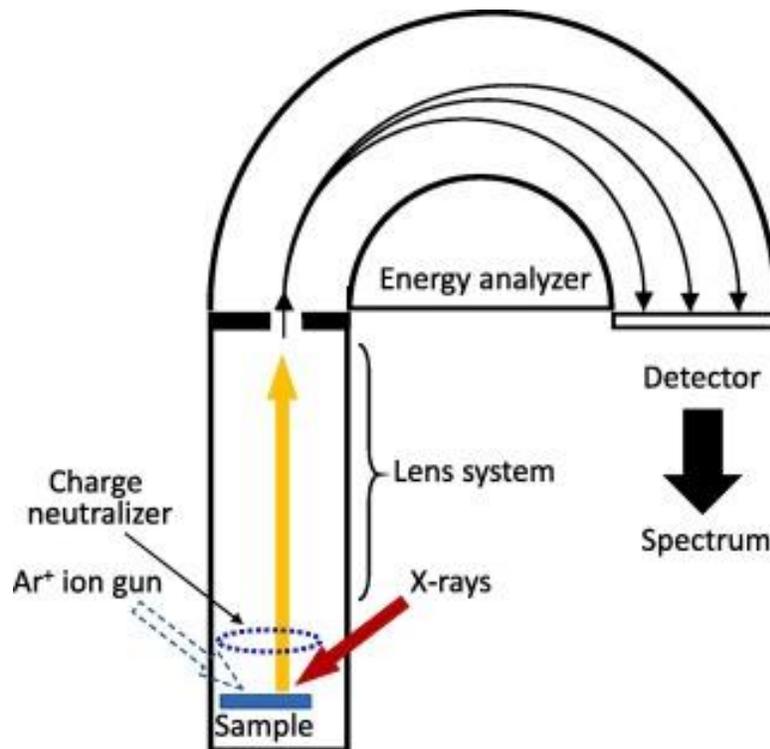


Figure 7: Construction of XPS

The electron spectrometer consists of following components.

- Source
- Sample Holder
- Analyser
- Detector
- Processor and The Read-Out

SOURCE:

The simple x-ray Photon source for X-ray photoelectron spectrum is x ray tube equipped with magnesium or aluminum metal target. Monochromator crystal can also provide having bandwidth of 0.3 eV. Much smaller spots on a surface to be examined.

SAMPLE HOLDER:

- Sample holder is located in between the source and the entrance slit of spectrometer. Crystal disperser selects the photon of known energy from the source and incident on the sample.
- The area inside the sample holder should be evacuated within 10⁻⁵ Torr. Pressure to avoid contamination of the surface sample.
- The gaseous sample can be introduced into a sample compartment through a slit, to provide a Pressure of 10⁻¹ torr.
- If the pressure is higher than attenuation of electron beam may take place, weaker signal may be obtained.

ANALYSER:

- It is hemispherical in shape with very high electrostatic field is applied on analyser. Pressure maintained inside the analyser is 10⁻⁵ torr.
- When the electron enters, into the hemispherical analyser, it travels in curved path and radius of Curvature depends upon magnitude of field and kinetic energy of the electron.

DETECTOR:

- The electron channel multiplier tube or transducer are required of X-Ray photoelectron Spectroscopy.
- When single electron pass through Glass tube coated with V/Pb the electron multiplier tube it is converted into number of electrons or pulses of electrons.

SIGNAL PROCESSOR AND READ OUT:

- The function of signal processor is to amplify the signal and read out device converts signal into spectrum.

5.8.4. WORKING:

The instrumentation of XPS system can be explained using the experimental set up as shown in figure. In a typical set up, when the sample kept in ultra-high vacuum is illuminated by the photons with energy ($h\nu$); soft x-rays, then after direct transfer of energy from the photons to the core level electrons their atoms on the surface emit electrons are called photoelectrons. The process can be explained as follows:

- A bound electron adsorbs a photon and converts part of the energy to kinetic energy.
- While leaving the atom, some energy of the electron is consumed in overcoming the Coulombic attraction of nucleus, thereby decreasing its kinetic energy.
- At this time the outer orbitals readjust, reducing the energy of the final state which is being created and delivering this extra energy to the outgoing electron.
- Then the XPS spectrum is obtained by determining the K.E. and number of electrons escaping from upper 0 to 10 nm of sample under investigation.
- XPS may be routinely employed to study inorganic compounds including metal alloys, semiconductors, polymers, glasses, ceramics, ion-modified materials, etc

5.8.5. APPLICATION OF XPS:

- Identification of active sites
- Determination of surface contamination on semiconductors
- Study of oxide layers on metals
- Analysis of dust on the sample
- Determination of Oxidation State all the elements of periodic table can be determined or identified except hydrogen and Helium, as they don't emit inner core electron

5.9 SCANNING ELECTRON MICROSCOPE (SEM):

SEM is a very powerful microscope which can magnify the sample by more than 100000 times. Optical microscopes are capable of magnifying only up to 1000 times and diffraction of the light limits the resolution. The wavelength of the visible light is in the 400 – 700 nm regions. Hence objects (or features) of smaller size cannot be seen using visible light. Scanning electron microscope is an improved model of an electron microscope

5.9.1 PRINCIPLE:

When the accelerated primary electrons strike the sample, it produces secondary electron. These secondary electrons are collected by a positively charged electron detector which in turn gives a three-dimensional image of the sample.

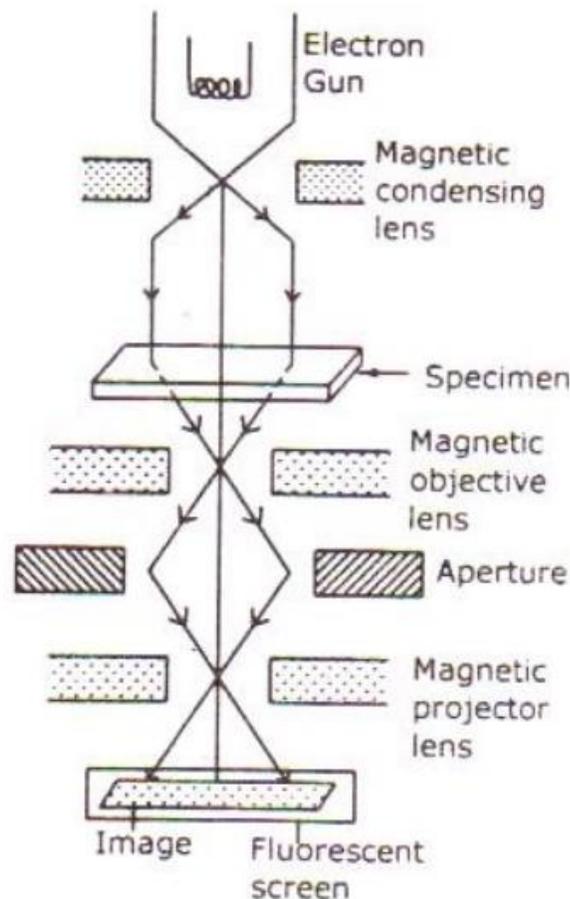


Figure8: Illustration of construction of SEM

5.9.2. CONSTRUCTION:

- An electron gun is used to produce high energy electrons.
- Two magnetic lenses are used as condenser lenses to convert the diverging electron beam into a fine beam of spot diameter of the order of a few nanometres.
- A scanning coil is used to detect the electron beam to scan the sample.
- The objective lens is used to focus the scanning beam on a desired spot on the sample.

- The intensities of secondary electron, back-scattered electrons and the X-rays are recorded using detectors. The images are then displayed on monitors

5.9.3 WORKING:

- when the high energy electron beam strikes the sample, some electrons are scattered due to elastic scattering (the back-scattered electrons), some electrons are knocked off from the surface (the secondary electrons) and some electrons penetrate deep into the inner shells of the sample atoms to knock off inner shell electrons due to which characteristic X-rays are produced.
- These are detected using detectors and the signals are amplified and displayed on a TV monitor. Samples are required to be conducting. Non-conducting samples are coated with a thin conducting material.
- SEM gives information about the surface features of the sample with resolution of the order of a few nanometers. This information can be used to study properties like reflectivity and roughness.

5.9.4 APPLICATIONS

- ❖ SEM images give information about the elements and compounds in the sample and their relative abundance. This is used to study properties like hardness and melting point.
- ❖ SEM is used to study biological specimens like pollen grains.
- ❖ SEM is used to study the corroded layers on metal surfaces.

5.6. TRANSMISSION ELECTRON MICROSCOPE (TEM):

5.6.1. INTRODUCTION:

We know in scanning electron microscope the resolution of the image is limited only up to 10 to 20 nm. This will not be useful to view the internal features of an atom or the morphology of a sample of size $\approx 0.2\text{nm}$. To examine the sample of size of 2 to 10 nm the transmission electron microscope can be used. In this microscope, the image is obtained by transmitting the electron through the specimen.

5.6.2 PRINCIPLE:

The working principle of the Transmission Electron Microscope (TEM) is like the light microscope. The major difference is that light microscopes use light rays to focus and produce an image while the TEM uses a beam of electrons to focus on the specimen, to produce an image. Electrons are made to pass through the specimen and the image is formed on the fluorescent screen, either by using the transmitted beam or by using the diffracted beam.

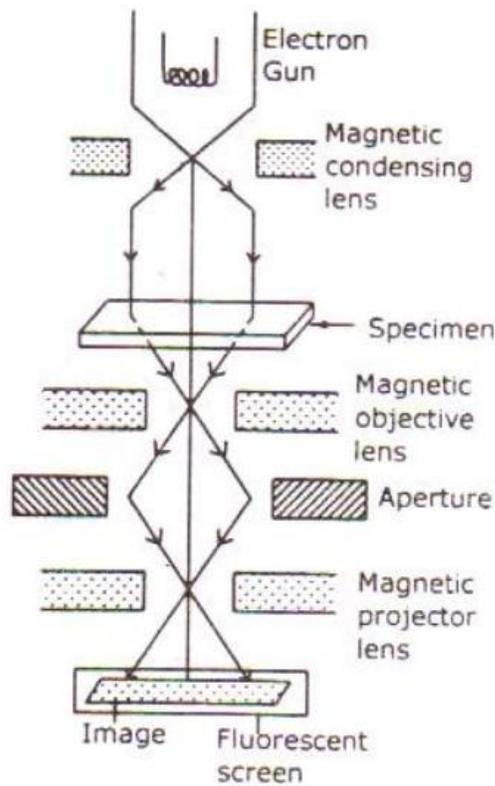


Figure 9: Illustration of construction of TEM

5.6.3 CONSTRUCTION:

- It consists of an electron gun to produce electron. Magnetic condensing lens is used to condense the electron and is used to adjust the size of the electron that fall onto the specimen.
- The specimen is placed in between the condensing lens and objective lens as shown in figure.
- The magnetic objective lens is used to block the high angle diffracted beam the aperture is used to eliminate the diffracted beam and in turn, it increases the contrast of the image.
- The magnetic projector lens is placed above the fluorescent screen in order to achieve high magnification. Can be recorded by using a fluorescent screen CCD charge coupled devices also.

5.6.4. WORKING:

- Stream of electrons is produced by the electron and is made to fall over the specimen using magnetic condensing lens. Based on the angle of incidence the Beam is partly transmitted and partly diffracted as shown in figure.
- Both the Magnetic ERL Condensing lens transmitted Beam and the diffracted beams are recombined at the E-WALLED SPHERE of reflection, which encloses all possible reflections from the Crystal are specimen satisfying the Bragg's law image as shown in figure. The combined image is called the phase contrast image.
- In order to increase the intensity and the contrast of the image and amplitude contrast image must be obtained for stop this can be achieved only by using the transmitting beam and does the diffracted beam has to be eliminated.
- Now in order to eliminate the diffracted beam that beam is passed through the magnetic objective lens and the aperture is shown in figure adjusted in such a way that the diffracted image is illuminated.
Thus, the final image being alone is passed through the projector lens for further magnification. Final image is recorded in the fluorescent screen or CCD this high contrast image is called Bright Field image.

5.6.5. APPLICATION:

- ❖ Transmission Electron Microscope is ideal for several different fields such as life sciences, nanotechnology, medical, biological, and material research, forensic analysis, gemmology, and metallurgy as well as industry and education.
- ❖ TEMs provide topographical, morphological, compositional, and crystalline information.
- ❖ The images allow researchers to view samples on a molecular level, making it possible to analyse structure and texture.
- ❖ This information is useful in the study of crystals and metals, but also has industrial applications.
- ❖ TEMs can be used in semiconductor analysis and production and the manufacturing of computer and silicon chips.